

Exercise Set 1

Problem 1. For $\kappa > 0$, show that the SLE hulls satisfy $\text{diam } K_t \rightarrow \infty$, $y_{\max}(t) \rightarrow \infty$, and $x_{\max}(t) \rightarrow \infty$ as $t \rightarrow \infty$.

Problem 2. With $X_t = g_t(x) - \sqrt{k}B_t$ and $x > 0$, show that $P[0 < X_t < b \text{ for all } t \geq 0] = 0$.

Problem 3. Show (directly from the definition) that $B_t^2 - t$ is a martingale.

Problem 4. Provide the details to the computation $P[B_{T_{a,b}^x} = a] = (x-b)/(a-b)$, using only optional sampling for bounded stopping times.

Problem 5. Use the definition of the Ito integral to show $\int_0^t B_s dB_s = 1/2 B_t^2 - 1/2 t$.

Due date : Friday, October 14.