University of Washington
Conformal Invariance and Probability - Math 583
S. Rohde Spring 2011

Exercise Set 1

Problem 1. If a_n is a sequence with $a_{n+m} \leq a_n + a_m$ for all $n, m \geq 0$, show that $\lim_{n\to\infty} \frac{a_n}{n}$ exists and equals $\inf_n \frac{a_n}{n}$. Conclude that $\lim b_n^{1/n}$ exists if b_n is a positive sequence with $b_{n+m} \leq b_n b_m$.

Problem 2. If $\alpha > 0$ and b_n is a real sequence with

$$\alpha b_{n+1}^2 + b_{n+1} \geq b_n$$

for all $n \geq 1$, show that

$$b_n \geq \frac{\min(b_1, 1/\alpha)}{n}.$$

Problem 3. Denoting b_n the number of bridges (excursions) on the hexagonal lattice, show that

$$\mu_{\text{bridge}} \equiv \lim b_n^{1/n} = \sqrt{2+\sqrt{2}}.$$

Due date : Monday, April 11.