University of Washington
Complex Analysis - Math 535
S. Rohde Winter 2018

Exercise Set 1

Problem 1: Solve those problems of the second half of the final exam for which you didn't receive 9 or 10 points.

Problem 2: Prove Montel's theorem in the unit disc using power series.

Problem 3: If f_n are analytic and locally uniformly bounded in a domain D, and if there is a sequence z_k of points in D with accumulation point in D such that

$$\lim_{n\to\infty} f_n(z_k) \quad \text{exists for all } k,$$

then f_n converges locally uniformly in D. Prove this!

Problem 4: Let D be a bounded domain and $f : D \to D$ be an analytic function. Consider the sequence $f_n = f \circ f \circ f \circ \cdots \circ f$ (n times) of iterates of f.

a) If f has an attractive fixpoint (in other words, if there is $z_0 \in D$ such that $|f'(z_0)| < 1$), show that f_n converges locally uniformly in D to this fixpoint.

b) If f has a fixpoint z_0 in D, then $|f'(z_0)| \leq 1$.

Problem 5: If f_n are meromorphic in a domain D and if f_n converge locally uniformly to a function $f : D \to \hat{\mathbb{C}}$, show that f is meromorphic or $f \equiv \infty$.

Problem 6: Construct a meromorphic function with simple poles of residue 1 at each point $p \in \mathbb{Z}^2$, and determine (with proof) the minimal degree of the Taylor polynomials for which your construction works.

Due date : Wedneday, January 17, before class.