Chapter 5

Problems

1. (a) $c\int_{-1}^{1}(1-x^2)dx = 1 \Rightarrow c = 3/4$

(b) $F(x) = \frac{3}{4}\int_{-1}^{x}(1-x^2)dx = \frac{3}{4}\left(x - \frac{x^3}{3} + \frac{2}{3}\right), -1 < x < 1$

2. $\int xe^{-x/2}dx = -2xe^{-x/2} - 4e^{-x/2}$. Hence,

$$c\int_{0}^{\infty} xe^{-x/2}dx = 1 \Rightarrow c = 1/4$$

$$P\{X > 5\} = \frac{1}{4}\int_{5}^{\infty} xe^{-x/2}dx = \frac{1}{4}[10e^{-5/2} + 4e^{-5/2}]$$

$$= \frac{14}{4}e^{-5/2}$$

3. No. $f(5/2) < 0$

4. (a) $\int_{20}^{\infty}\frac{10}{x^2}dx = \frac{-10}{x}\Big|_{20}^{\infty} = 1/2$.

(b) $F(y) = \int_{10}^{y}\frac{10}{x^2}dx = 1 - \frac{10}{y}$, $y > 10$. $F(y) = 0$ for $y < 10$.

(c) $\sum_{i=3}^{6}\binom{6}{i}\left(\frac{2}{3}\right)^i\left(\frac{1}{3}\right)^{6-i}$ since $\overline{F}(15) = \frac{10}{15}$. Assuming independence of the events that the devices exceed 15 hours.

5. Must choose c so that

$$.01 = \int_{c}^{1} 5(1-x)^4 dx = (1-c)^5$$

so $c = 1 - (.01)^{1/5}$.

6. (a) $E[X] = \frac{1}{4}\int_0^\infty x^2 e^{-x/2}dx = 2\int_0^\infty y^2 e^{-y}dx = 2\Gamma(3) = 4$

(b) By symmetry of $f(x)$ about $x = 0$, $E[X] = 0$

(c) $E[X] = \int_5^\infty \frac{5}{x}dx = \infty$

7. $\int_0^1 (a + bx^2)dx = 1$ or $a + \frac{b}{3} = 1$

$\int_0^1 x(a + bx^2)dx = \frac{3}{5}$ or $\frac{a}{2} + \frac{b}{4} = 3/5$. Hence,

$a = \frac{3}{5}$, $b = \frac{6}{5}$

8. $E[X] = \int_0^\infty x^2 e^{-x}dx = \Gamma(3) = 2$

9. If s units are stocked and the demand is X, then the profit, $P(s)$, is given by

$$P(s) = bX - (s - X)P \qquad \text{if } X \le s$$
$$= sb \qquad \text{if } X > s$$

Hence

$$E[P(s)] = \int_0^s (bx - (s - x)\ell) f(x)dx + \int_s^\infty sbf(x)dx$$
$$= (b + \ell)\int_0^s xf(x)dx - s\ell\int_0^s f(x)dx + sb\left[1 - \int_0^s f(x)dx\right]$$
$$= sb + (b + \ell)\int_0^s (x - s) f(x)dx$$

Differentiation yields

$$\frac{d}{ds}E[P(s)] = b + (b + \ell)\frac{d}{ds}\left[\int_0^s xf(x)dx - s\int_0^s f(x)dx\right]$$
$$= b + (b + \ell)\left[sf(s) - sf(s) - \int_0^s f(s)dx\right]$$
$$= b - (b + \ell)\int_0^s f(x)dx$$

Equating to zero shows that the maximal expected profit is obtained when s is chosen so that

$$F(s) = \frac{b}{b+\ell}$$

where $F(s) = \int_0^s f(x)dx$ is the cumulative distribution of demand.

10. (a) $P\{\text{goes to } A\} = P\{5 < X < 15 \text{ or } 20 < X < 30 \text{ or } 35 < X < 45 \text{ or } 50 < X < 60\}$.
$= 2/3$ since X is uniform (0, 60).

(b) same answer as in (a).

11. X is uniform on $(0, L)$.

$$P\left\{\min\left(\frac{X}{L-X}, \frac{L-X}{X}\right) < 1/4\right\}$$
$$= 1 - P\left\{\min\left(\frac{X}{L-X}, \frac{L-X}{X}\right) > 1/4\right\}$$
$$= 1 - P\left\{\frac{X}{L-X} > 1/4, \frac{L-X}{X} > 1/4\right\}$$
$$= 1 - P\{X > L/5, X < 4L/5\}$$
$$= 1 - P\left\{\frac{L}{5} < X < 4L/5\right\}$$
$$= 1 - \frac{3}{5} = \frac{2}{5}.$$

13. $P\{X > 10\} = \frac{2}{3}$, $P\{X > 25 \mid X > 15\} = \frac{P\{X > 25\}}{P\{X > 15\}} = \frac{5/30}{15/30} = 1/3$
where X is uniform (0, 30).

14. $E[X^n] = \int_0^1 x^n dx = \frac{1}{n+1}$

$P\{X^n \le x\} = P\{X \le x^{1/n}\} = x^{1/n}$

$E[X^n] = \int_0^1 x \frac{1}{n} x^{\left(\frac{1}{n}-1\right)} dx = \frac{1}{n}\int_0^1 x^{1/n} dx = \frac{1}{n+1}$

15. (a) $\Phi(.8333) = .7977$
(b) $2\Phi(1) - 1 = .6827$
(c) $1 - \Phi(.3333) = .3695$
(d) $\Phi(1.6667) = .9522$
(e) $1 - \Phi(1) = .1587$

16. $P\{X > 50\} = P\left\{\frac{X-40}{4} > \frac{10}{4}\right\} = 1 - \Phi(2.5) = 1 - .9938$

Hence, $(P\{X < 50\})^{10} = (.9938)^{10}$

17. E[Points] = 10(1/10) + 5(2/10) + 3(2/10) = 2.6

18. $.2 = P\left\{\frac{X-5}{\sigma} > \frac{9-5}{\sigma}\right\} = P\{Z > 4/\sigma\}$ where Z is a standard normal. But from the normal table $P\{Z < .84) \approx .80$ and so

$.84 \approx 4/\sigma$ or $\sigma \approx 4.76$

That is, the variance is approximately $(4.76)^2 = 22.66$.

19. Letting $Z = (X - 12)/2$ then Z is a standard normal. Now, $.10 = P\{Z > (c - 12)/2\}$. But from Table 5.1, $P\{Z < 1.28\} = .90$ and so

$(c - 12)/2 = 1.28$ or $c = 14.56$

20. Let X denote the number in favor. Then X is binomial with mean 65 and standard deviation $\sqrt{65(.35)} \approx 4.77$. Also let Z be a standard normal random variable.

(a) $P\{X \geq 50\} = P\{X \geq 49.5\} = P\{X - 65\}/4.77 \geq -15.5/4.77$
$\approx P\{Z \geq -3.25\} \approx .9994$

(b) $P\{59.5 \leq X \leq 70.5\} \approx P\{-5.5/4.77 \leq Z \leq 5.5/4.77\}$
$= 2P\{Z \leq 1.15\} - 1 \approx .75$

(c) $P\{X \leq 74.5\} \approx P\{Z \leq 9.5/4.77\} \approx .977$

22. (a) $P\{.9000 - .005 < X < .9000 + .005\}$

$= P\left\{-\frac{.005}{.003} < Z < \frac{.005}{.003}\right\}$

$= P\{-1.67 < Z < 1.67\}$

$= 2\Phi(1.67) - 1 = .9050.$

Hence 9.5 percent will be defective (that is each will be defective with probability $1 - .9050 = .0950$).

(b) $P\left\{-\frac{.005}{\sigma} < Z < \frac{.005}{\sigma}\right\} = 2\Phi\left(\frac{.005}{\sigma}\right) - 1 = .99$ when

$$\Phi\left(\frac{.005}{\sigma}\right) = .995 \Rightarrow \frac{.005}{\sigma} = 2.575 \Rightarrow \sigma = .0019\ .$$

23. (a) $P\{149.5 < X < 200.5\} = P\left\{\dfrac{149.5 - \frac{1000}{6}}{\sqrt{1000\frac{1}{6}\frac{5}{6}}} < Z < \dfrac{200.5 - \frac{1000}{6}}{\sqrt{1000\frac{1}{6}\frac{5}{6}}}\right\}$

$$= \Phi\left(\frac{200.5 - 166.7}{\sqrt{5000/36}}\right) - \Phi\left(\frac{149.5 - 166.7}{\sqrt{5000/36}}\right)$$

$$\approx \Phi(2.87) + \Phi(1.46) - 1 = .9258.$$

(b) $P\{X < 149.5\} = P\left\{Z < \dfrac{149.5 - 800(1/5)}{\sqrt{800\frac{1}{5}\frac{4}{5}}}\right\}$

$$= P\{Z < -.93\}$$
$$= 1 - \Phi(.93) = .1762.$$

24. With C denoting the life of a chip, and ϕ the standard normal distribution function we have

$$P\{C < 1.8 \times 10^6\} = \phi\left(\frac{1.8\times 10^6 - 1.4\times 10^6}{3\times 10^5}\right)$$
$$= \phi(1.33)$$
$$= .9082$$

Thus, if N is the number of the chips whose life is less than 1.8×10^6 then N is a binomial random variable with parameters (100, .9082). Hence,

$$P\{N > 19.5\} \approx 1 - \phi\left(\frac{19.5 - 90.82}{90.82(.0918)}\right) = 1 - \phi(-24.7) \approx 1$$

25. Let X denote the number of unacceptable items among the next 150 produced. Since X is a binomial random variable with mean 150(.05) = 7.5 and variance 150(.05)(.95) = 7.125, we obtain that, for a standard normal random variable Z.

$$P\{X \le 10\} = P\{X \le 10.5\}$$
$$= P\left\{\frac{X - 7.5}{\sqrt{7.125}} \le \frac{10.5 - 7.5}{\sqrt{7.125}}\right\}$$
$$\approx P\{Z \le 1.1239\}$$
$$= .8695$$

The exact result can be obtained by using the text diskette, and (to four decimal places) is equal to .8678.

27. $P\{X > 5{,}799.5\} = P\left\{Z > \dfrac{799.5}{\sqrt{2{,}500}}\right\}$

$$= P\{Z > 15.99\} = \text{negligible}.$$

28. Let X equal the number of lefthanders. Assuming that X is approximately distributed as a binomial random variable with parameters $n = 200$, $p = .12$, then, with Z being a standard normal random variable,

$$P\{X > 19.5\} = P\left\{\frac{X - 200(.12)}{\sqrt{200(.12)(.88)}} > \frac{19.5 - 200(.12)}{\sqrt{200(.12)(.88)}}\right\}$$
$$\approx P\{Z > -.9792\}$$
$$\approx .8363$$

29. Let s be the initial price of the stock. Then, if X is the number of the 1000 time periods in which the stock increases, then its price at the end is

$$su^X d^{1000\text{-}X} = sd^{1000}\left(\frac{u}{d}\right)^X$$

Hence, in order for the price to be at least 1.3s, we would need that

$$d^{1000}\left(\frac{u}{d}\right)^X > 1.3$$

or

$$X > \frac{\log(1.3) - 1000\log(d)}{\log(u/d)} = 469.2$$

That is, the stock would have to rise in at least 470 time periods. Because X is binomial with parameters 1000, .52, we have

$$P\{X > 469.5\} = P\left\{\frac{X - 1000(.52)}{\sqrt{1000(.52)(.48)}} > \frac{469.5 - 1000(.52)}{\sqrt{1000(.52)(.48)}}\right\}$$
$$\approx P\{Z > -3.196\}$$
$$\approx .9993$$

30. $$P\{\text{in black}\} = \frac{P\{5 \mid \text{black}\}\alpha}{P\{5 \mid \text{black}\}\alpha + P\{5 \mid \text{white}\}(1-\alpha)}$$

$$= \frac{\frac{1}{2\sqrt{2\pi}} e^{-(5-4)^2/8}\alpha}{\frac{1}{2\sqrt{2\pi}} e^{-(5-4)^2/8}\alpha + (1-\alpha)\frac{1}{3\sqrt{2\pi}} e^{-(5-6)^2/18}}$$

$$= \frac{\frac{\alpha}{2} e^{-1/8}}{\frac{\alpha}{2} e^{-1/8} + \frac{(1-\alpha)}{3} e^{-1/8}}$$

α is the value that makes preceding equal 1/2

31. (a) $E[|X-a|] = \int_a^A (x-a)\frac{dx}{A} + \int_0^a (a-x)\frac{dx}{A} = \frac{A}{2} - \left(a - \frac{a^2}{A}\right)$

$$\frac{d}{da}(\quad) = \frac{2a}{A} - 1 = 0 \Rightarrow a = A/2$$

(b) $E[|X-a|] = \int_0^a (a-x)\lambda e^{-\lambda x}dx + \int_a^\infty (x-a)\lambda e^{-\lambda x}dx$

$$= a(1-e^{-\lambda a}) + ae^{-\lambda a} + \frac{e^{-\lambda a}}{\lambda} - \frac{1}{\lambda} + ae^{-\lambda a} + \frac{e^{-\lambda a}}{\lambda} - ae^{-\lambda a}$$

Differentiation yields that the minimum is attained at $\bar{a}$ where

$e^{-\lambda \bar{a}} = 1/2$ or $\bar{a} = \log 2/\lambda$

(c) Minimizing a = median of F

32. (a) e^{-1}
(b) $e^{-1/2}$

33. e^{-1}

34. (a) $P\{X > 20\} = e^{-1}$

(b) $P\{X > 30 \mid X > 10 = \frac{P\{X > 30\}}{P\{X > 10\}} = \frac{1/4}{3/4} = 1/3$

35. (a) $\exp\left[-\int_{40}^{50} \lambda(t)dt\right] = e^{-.35}$

(b) $e^{-1.21}$

36. (a) $1 - F(2) = \exp\left[-\int_0^2 t^3 dt\right] = e^{-4}$

(b) $\exp[-(.4)^4/4] - \exp[-(1.4)^4/4]$

(c) $\exp\left[-\int_1^2 t^3 dt\right] = e^{-15/4}$

37. (a) $P\{|X| > 1/2\} = P\{X > 1/2\} + P\{X < -1/2\} = 1/2$

(b) $P\{|X| \le a\} = P\{-a \le X \le a\} = a$, $0 < a < 1$. Therefore,
$f_{|X|}(a) = 1,\ 0 < a < 1$
That is, $|X|$ is uniform on (0, 1).

38. For both roots to be real the discriminant $(4Y)^2 - 44(Y+2)$ must be ≥ 0. That is, we need that $Y^2 \geq Y + 2$. Now in the interval $0 < Y < 5$.

$$Y^2 \geq Y + 2 \Leftrightarrow Y \geq 2 \text{ and so}$$
$$P\{Y^2 \geq Y + 2\} = P\{Y \geq 2\} = 3/5.$$

39. $F_Y(y) = P\{\log X \leq y\}$
$= P\{X \leq e^y\} = F_X(e^y)$

$f_Y(y) = f_X(e^y)e^y = e^y e^{-e^y}$

40. $F_Y(y) = P\{e^X \leq y\}$
$= F_X(\log y)$

$f_Y(y) = f_X(\log y)\dfrac{1}{y} = \dfrac{1}{y}, \ 1 < y < e$

Theoretical Exercises

1. The integration by parts formula $\int u dv = uv - \int v du$ with $dv = -2bxe^{-bx^2}$, $u = -x/2b$ yields that

$$\int_0^\infty x^2 e^{-bx^2} dx = \frac{-xe^{-bx^2}}{2b}\Big|_0^\infty + \frac{1}{2b}\int_0^\infty e^{-bx^2} dx$$
$$= \frac{1}{(2b)^{3/2}}\int_0^\infty e^{-y^2/2} dy \quad \text{by } y = x\sqrt{2b}$$
$$= \frac{\sqrt{2\pi}}{2}\frac{1}{(2b)^{3/2}} = \frac{\sqrt{\pi}}{4b^{3/2}}$$

where the above uses that $\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-y^2/2} dy = 1/2$. Hence, $a = \frac{4b^{3/2}}{\sqrt{\pi}}$

2. $$\int_0^\infty P\{Y < -y\} dy = \int_0^\infty \int_{-\infty}^{-y} f_Y(x) dx\, dy$$
$$= \int_{-\infty}^0 \int_0^{-x} f_Y(x) dy\, dx = -\int_{-\infty}^0 x f_Y(x)\, dx$$

Similarly,

$$\int_0^\infty P\{Y > y\} dy = \int_0^\infty x f_Y(x)\, dx$$

Subtracting these equalities gives the result.

4. $$E[aX + b] = \int (ax + b) f(x)\, dx = a\int x f(x) dx + b\int f(x) dx$$
$$= aE[X] + b$$

5. $$E[X^n] = \int_0^\infty P\{X^n > t\} dt$$
$$= \int_0^\infty P\{X^n > x^n\} nx^{n-1} dx \quad \text{by } t = x^n,\ dt = nx^{n-1}dx$$
$$= \int_0^\infty P\{X > x\} nx^{n-1} dx$$

6. Let X be uniform on (0, 1) and define E_a to be the event that X is unequal to a. Since $\bigcap_a E_a$ is the empty set, it must have probability 0.

7. $SD(aX+b) = \sqrt{\text{Var}(aX+b)} = \sqrt{a^2\sigma^2} = |a|\sigma$

8. Since $0 \le X \le c$, it follows that $X^2 \le cX$. Hence,

$$\begin{aligned}\text{Var}(X) &= E[X^2] - (E[X])^2 \\ &\le E[cX - (E[X])^2 \\ &= cE[X] - (E[X])^2 \\ &= E[X](c - E[X]) \\ &= c^2[\alpha(1-\alpha)] \quad \text{where } \alpha = E[X]/c \\ &\le c^2/4\end{aligned}$$

where the last inequality first uses the hypothesis that $P\{0 \le X \le c\} = 1$ to calculate that $0 \le \alpha \le 1$ and then uses calculus to show that $\underset{0\le\alpha\le1}{\text{maximum}}\ \alpha(1-\alpha) = 1/4$.

9. The final step of parts (a) and (b) use that $-Z$ is also a standard normal random variable.

(a) $P\{Z > x\} = P\{-Z < -x\} = P\{Z < -x\}$

(b) $P\{|Z| > x\} = P\{Z > x\} + P\{Z < -x\} = P\{Z > x\} + P\{-Z > x\}$
$= 2P\{Z > x\}$

(c) $P\{|Z| < x\} = 1 - P\{|Z| > x\} = 1 - 2P\{Z > x\}$ by (b)
$= 1 - 2(1 - P\{Z < x\})$

10. With $c = 1/\left(\sqrt{2\pi}\sigma\right)$ we have

$f(x) = ce^{-(x-\mu)^2/2\sigma^2}$

$f'(x) = -ce^{-(x-\mu)^2/2\sigma^2}(x-\mu)/\sigma^2$

$f''(x) = c\sigma^{-4}e^{-(x-\mu)^2/2\sigma^2}(x-\mu)^2 - c\sigma^{-2}e^{-(x-\mu)^2/2\sigma^2}$

Therefore,

$f''(\mu+\sigma) = f''(\mu-\sigma) = c\sigma^{-2}e^{-1/2} - c\sigma^{-2}e^{-1/2} = 0$

11. $E[X^2] = \int_0^\infty P\{X > x\}2x^{2-1}dx = 2\int_0^\infty xe^{-\lambda x}dx = \frac{2}{\lambda}E[X] = 2/\lambda^2$

12. (a) $\frac{b+a}{2}$

(b) μ

(c) $1 - e^{-\lambda m} = 1/2$ or $m = \frac{1}{\lambda}\log 2$

13. (a) all values in (a, b)

(b) μ

(c) 0

14. $P\{cX < x\} = P\{X < x/c\} = 1 - e^{-\lambda x/c}$

15. $\lambda(t) = \dfrac{f(t)}{\overline{F}(t)} = \dfrac{1/a}{(a-t)/a} = \dfrac{1}{a-t}$, $0 < t < a$

16. If X has distribution function F and density f, then for $a > 0$

$$F_{aX}(t) = P\{aX \le t\} = F(t/a)$$

and

$$f_{ax} = \frac{1}{a} f(t/a)$$

Thus,

$$\lambda_{aX}(t) = \frac{\frac{1}{a} f(t/a)}{1 - F(t/a)} = \frac{1}{a}\lambda_X(t/a)\,.$$

18. $E[X^k] = \int_0^\infty x^k \lambda e^{-\lambda x} dx \; = \; \lambda^{-k} \int_0^\infty \lambda e^{-\lambda x} (\lambda x)^k \, dx$

$$= \lambda^{-k}\,\Gamma(\mathrm{k}+1) = \mathrm{k}!/\lambda^k$$

19. $E[X^k] = \dfrac{1}{\Gamma(t)} \displaystyle\int_0^\infty x^k \lambda e^{-\lambda x} (\lambda x)^{t-1} dx$

$$= \frac{\lambda^{-k}}{\Gamma(t)} \int_0^\infty \lambda e^{-\lambda x} (\lambda x)^{t+k-1} dx$$

$$= \frac{\lambda^{-k}}{\Gamma(t)} \Gamma(t+k)$$

Therefore,

$E[X] = t/\lambda$,
$E[X^2] = = \lambda^{-2}\Gamma(t+2)/\Gamma(t) = (t+1)t/\lambda^2$

and thus

$$\mathrm{Var}(X) = (t+1)t/\lambda^2 - t^2/\lambda^2 = t/\lambda^2$$

20. $\Gamma(1/2) = \int_0^\infty e^{-x}x^{-1/2}dx$

$= \sqrt{2}\int_0^\infty e^{-y^2/2}dy$ by $x = y^2/2$, $dx = ydy = \sqrt{2x}\ dy$

$= 2\sqrt{\pi}\int_0^\infty (2\pi)^{-1/2}e^{-y^2/2}dy$

$= 2\sqrt{\pi}P\{Z > 0\}$ where Z is a standard normal

$= \sqrt{\pi}$

21. $1/\lambda(s) = \int_{x \ge s} \lambda e^{-\lambda x}(\lambda x)^{t-1}dx / \lambda e^{-\lambda s}(\lambda s)^{t-1}$

$= \int_{x \ge s} e^{-\lambda(x-s)}(x/s)^{t-1}dx$

$= \int_{y \ge 0} e^{-\lambda y}(1 + y/s)^{t-1}dy$ by letting $y = x - s$

As the above, equal to the inverse of the hazard rate function, is clearly decreasing in s when $t \ge 1$ and increasing when $t \le 1$ the result follows.

22. $\lambda(s) = c(s - v)^{\beta-1}$, $s > v$ which is clearly increasing when $\beta \ge 1$ and decreasing otherwise.

23. $F(\alpha) = 1 - e^{-1}$

24. Suppose X is Weibull with parameters v, α, β. Then

$$P\left\{\left(\frac{X-v}{\alpha}\right)^\beta \le x\right\} = P\left\{\frac{X-v}{\alpha} \le x^{1/\beta}\right\}$$
$$= P\{X \le v + \alpha x^{1/\beta}\}$$
$$= 1 - \exp\{-x\}.$$

25. We use Equation (6.3).

$$E[X] = B(a + 1, b)/B(A, b) = \frac{\Gamma(a+1)}{\Gamma(a+b+1)}\frac{\Gamma(a+b)}{\Gamma(a)} = \frac{a}{a+b}$$

$$E[X^2] = B(a + 2, b)/B(a, b) = \frac{\Gamma(a+2)}{\Gamma(a+b+2)}\frac{\Gamma(a+b)}{\Gamma(a)} = \frac{(a+1)a}{(a+b+1)(a+b)}$$

Thus,

$$\text{Var}(X) = \frac{(a+1)a}{(a+b+1)(a+b)} - \frac{a^2}{(a+b)^2} = \frac{ab}{(a+b+1)(a+b)^2}$$

26. $(X - a)/(b - a)$

28. $P\{F(X \le x\} = P\{X \le F^{-1}(x)\}$
$= F(F^{-1}(x))$
$= x$

29. $F_Y(x) = P\{aX + b \le x\}$

$$= P\left\{X \le \frac{x-b}{a}\right\} \text{ when } a > 0$$

$= F_X((x - b)/a)$ when $a > 0$.

$$f_Y(x) = \frac{1}{a} f_X((x-b)/a) \quad \text{if } a > 0.$$

When $a < 0$, $F_Y(x) = P\left\{X \ge \frac{x-b}{a}\right\} = 1 - F_X\left(\frac{x-b}{a}\right)$ and so

$$f_Y(x) = -\frac{1}{a} f_X\left(\frac{x-b}{a}\right).$$

30. $F_Y(x) = P\{e^X \le x\}$
$= P\{X \le \log x\}$
$F_X(\log x)$

$f_Y(x) = f_X(\log x)/x$

$$= \frac{1}{x\sqrt{2\pi}\sigma} e^{-(\log x - \mu)^2 / 2\sigma^2}$$