Kolya and Avi's Talk

Andrew You

July 16, 2015

Kolya: ***Graph Minor Theorem***

Definition 1 (Minor)**.** *G_1 is a minor of G_2 if G_1 can be obtained from G_2 by deletions and contractions.*

Consider $f : V_2 \to V_1$. If $U \sim V$ in G_1, then $u' \sim v'$ in G_2 for some $u', v' \in V_2$ i.e. $f(u') = u$ and $f(v') = v$. A graph G is a forest (no cycles) if and only if K_3 is not a minor of G.

Theorem 1 (Wagner's Theorem)**.** *A graph is planar if and only if it does not contain K_3 or $K_{3,3}$ as minors.*

Definition 2 (Outerplanar)**.** *A graph is outerplanar if you can draw it in the plane with all vertices on the outer free.*

Definition 3 (Closed)**.** *Suppose G is an arbitrary set of graphs. $H \subset G$ is closed under minors if whenever $G_1 \in H$ and G_2 is a minor of G_1, then $G_2 \in H$.*

Remark 1 (Notation)**.** *$G_2 \trianglelefteq G_1$ if G_2 is a minor of G_1.*

Theorem 2 (Graph Minor Theorem)**.** *$\trianglelefteq$ is a well-partial order. Every $H \in G$ closed under minors has the form $H = \{G : G$ does not have any of $G_1, \ldots, G_n$ as minor.$\}$*

Consider a directed graph, $D = (V, A)$ where A is the set of adjacent edges. Then, D is called weakly connected if it is connected as an undirected graph. D is strongly connected if every vertex can be reached from every other. D_1 is a minor of D_2 if it can be obtained by deletions and cycle contractions.

Definition 4. *Consider $D = (V, A)$, where $c : A \to \mathbb{R}^+$ and $s, t \in V$. A flow from s to t is $f : A \to \mathbb{R}^+$ such that*

1. $\forall a \in A$, $f(a) \leq c(a)$
2. $\forall v \neq s, t$, $\sum_{a \text{ out of } v} f(a) = \sum_{a \text{ into } v} f(a)$.

Avi: ***Degeneracy in the Discrete Inverse Problem***

Given a graph with boundary, G, the boundary graph is as follows: $\partial G = (\partial V, \partial E)$ where $\partial E = \{(v, w) :$ $\exists$ a path through the interior from v to $w\}$. A response map is defined as $L_G : \Gamma \rightarrowtail \partial\Gamma$.

Definition 5 (Fibers of the Response Map)**.** *Given a boundary network $\partial\Gamma$, the fiber over $\partial\Gamma$ is the set $L_G^{-1}(\partial\Gamma)$.*

Definition 6 (N-to-1)**.** *A graph is N-to-1 if*

1. *$\exists$ a fiber of size N.*
2. *Every fiber has size $\leq N$.*

In general, $\kappa(G) = \sup_{\partial\Gamma} |L_G^{-1}(\partial\Gamma)|$. In addition, $\kappa(G) = N$ if and only if G is N-to-1. Ernie Esser discovered that for Triangle-in-triangle graphs, $\mathcal{T}$, $\kappa(\mathcal{T}) = 2$. Quad and switch graps are both graphs with flow.

Definition 7 (Strip Graphs)**.** *A strip is the result of obtaining a sequence of quads and switches, such that there are no internal switches.*