MAKE THEM FULL RANK

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1. Omega Matrix

Definition 1.1. An *Omega Matrix* is a response matrix Λ with n boundary nodes added by an all-ones matrix which multiplied by a scalar $\frac{\lambda}{n}$, denoted as $\Omega_\lambda(\Lambda)$.

Lemma 1.2. *λ is an eigenvalue of $\Omega_\lambda(\Lambda)$ with eigenvector* $L = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$.

Lemma 1.3. *Suppose a response matrix Λ is from a connected all-positive-conductivity network, or Λ has nullity 1, then $\Omega_\lambda(\Lambda)$ has full rank if and only if λ is not 0.*

Corollary 1.4. *Given the response matrix Λ and a set of currents I on the boundary, suppose Λ has rank $n-1$, then possible corressponding voltages are* $V = \Omega_1(\Lambda)^{-1}I + kL$, *where k is an arbitrary scalar and* $L = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$.

Proof. Since Λ has nullity 1 (see [1]), the nullspace of Λ, say N, is span$\{L\}$. Let C be the orthogonal complement of N. It's easy to see that $\Omega_\lambda(\Lambda)$ maps C onto C and N onto N. This directly leads to the lemmas above. Notice that C, as a range, is the collection of all possible boundary current vectors. Therefore $\Omega_\lambda(\Lambda)$ maps any voltage vector $\in C$ to the correct current vector, and maps other voltage vectors to their corressponding current vectors with a difference of L multiplied by some non-zero scalar. This gives a proof for the corollary. $\square$

References

[1] Addington, Nicolas. "Stars, Eigenvalues, and Negative Conductivities"; 2003.

Date: August 2003.