Recovery of Circular Annular Networks

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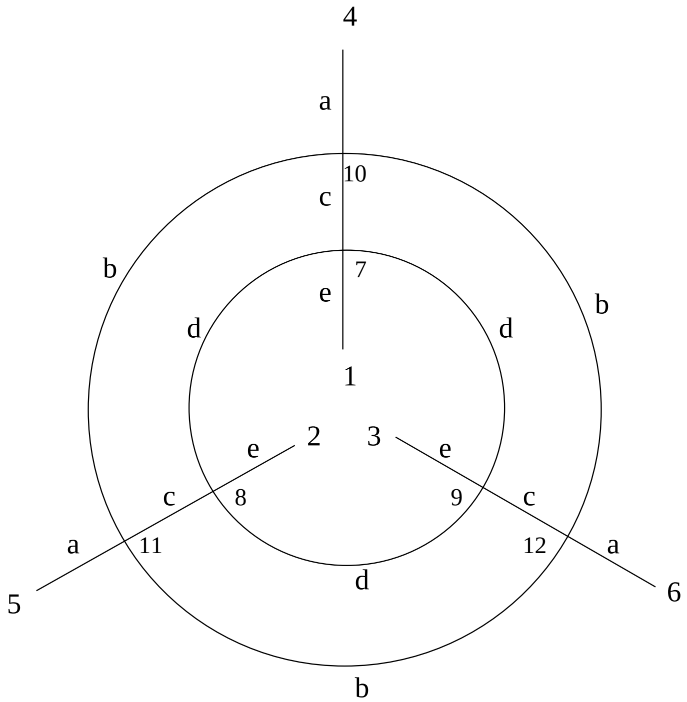
The intent of this research was to prove the conjecture that the map $\gamma \rightarrow \Lambda$ is infinite to one. [1] To test this, we start with a known K, and then assign an arbitrary value α to one of the boundary conductors. With this boundary condition known, we can thus solve the values of the other conductors as a set of parametric equations. Using these parametric equations as values in K, we generate a new Λ, which is then compared to the original Λ. If the value of α is within the range of possible values given K, then the two values of Λ should be identical, proving that $\gamma \rightarrow \Lambda$ is infinite to one. We will then examine this problem in the case of semi-conductor networks, along with the general properties of these semiconductors.



1 General Recovery of a Layered 3 ray, 2 ring network

First, we obtain the Λ matrix for a layered network. This is done by taking $\Lambda = A - B \cdot C^{-1} \cdot B'$ where

$$A = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & e & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & e \end{bmatrix}$$

$$B = \begin{bmatrix} -a & 0 & 0 & 0 & 0 & 0 \\ 0 & -a & 0 & 0 & 0 & 0 \\ 0 & 0 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & -e & 0 & 0 \\ 0 & 0 & 0 & 0 & -e & 0 \\ 0 & 0 & 0 & 0 & 0 & -e \end{bmatrix}$$

$$C = \begin{bmatrix} a+2b+c & -b & -b & -c & 0 & 0 \\ -b & a+2b+c & -b & 0 & -c & 0 \\ -b & -b & a+2b+c & 0 & 0 & -c \\ -c & 0 & 0 & c+2d+e & -d & -d \\ 0 & -c & 0 & -d & c+2d+e & -d \\ 0 & 0 & -c & -d & -d & c+2d+e \end{bmatrix}$$

This will give us our response matrix Λ

$$\Lambda = \begin{bmatrix} \Sigma & \alpha & \alpha & \beta & \gamma & \gamma \\ \alpha & \Sigma & \alpha & \gamma & \beta & \gamma \\ \alpha & \alpha & \Sigma & \gamma & \gamma & \beta \\ \beta & \gamma & \gamma & \Pi & \delta & \delta \\ \gamma & \beta & \gamma & \delta & \Pi & \delta \\ \gamma & \gamma & \beta & \delta & \delta & \Pi \end{bmatrix}$$

The validity of α can be determined by taking the signs of the conductors given by the parametric equations. All of these must be positive, as all conductors have a positive conductance. This is also important due to the frequent divisions, which clearly must not be divisions by zero. However, there are still an infinite number of such α, thus satisfying the first half of the conjecture.

The second half of the conjecture requires that all $\gamma_{i,j}$ resulting from a valid α result in indistinguishable response matrices Λ. In order to do so, we must first symbolically derive the parametric equations given arbitrary values for the conductances. When we then place these parametric equations into K, we can then derive Λ, which can be verified to be the same as the Λ we began with, proving the second half of the conjecture.

In order to derive the parametric equations, we run the following program in Mathematica, derived from Matlab code found in [1]:

```
<<LinearAlgebra‘MatrixManipulation‘
  Clear[a, b, c, d, e, s, t, u, v, w, F, G, H, J, K, X, Y, Z]


  U1 = LinearSolve[{{bet, gam, gam}, {gam, bet, gam}, {gam, gam, bet}},
  ({{-pie}, {-delt}, {-delt}} + {{t}, {0},{0}})]
  U11 = Take[U1, 1]
  U12 = Take[U1, {2, 2}]
  U13 = Take[U1, {3, 3}]
  gam2o8 = (1/U12)*(gam) + U11*(alph) + U12*(sig) + U13*(alph)
  gam3o9 = (1/U13)*(gam) + U11*(alph) + U12*(alph) +U13*(sig)
  gam2o8=Det[gam2o8]
  gam3o9=Det[gam3o9]
```

```
U2 = LinearSolve[{{bet, gam, gam}, {gam, bet, gam}, {gam, gam, bet}},
({{-alph}, {-sig}, {-alph}} + {{0}, {gam2o8}, {0}})]
U21 = Take[U2, {1, 1}]
U22 = Take[U2, {2, 2}]
U23 = Take[U2, {3, 3}]
gam6o12 = (1/U23)*(gam) + U21*(delt)  + U22*(delt) + U23*(pie)
gam6o12=Det[gam6o12]

U3 = LinearSolve[{{bet, gam, gam}, {gam, bet, gam}, {gam, gam, bet}},
({{alph}, {alph}, {sig}} + {{0}, {0}, {gam3o9}})]
U31 = Take[U3, {1, 1}]
U32 = Take[U3, {2, 2}]
U33 = Take[U3, {3, 3}]
gam5o11 = (1/U32)*(delt) + U31*(delt) + U32*(pie) + U33*(delt)
z12 = (1/gam6o12)*(U33*gam6o12 - bet) - (U31*delt) -(U32*delt) - (U33*pie)
gam9o12 = -gam3o9/z12
gam11o12 = (-1/z12)*(gam + U31*delt + U32*pie + U33*delt)
gam10o12 = (-1/z12)*(gam + U31*pie + U32*delt + U33*delt)
gam5o11=Det[gam5o11]
gam9o12=Det[gam9o12]
gam11o12=Det[gam11o12]
gam10o12=Det[gam10o12]

U4 = LinearSolve[{{bet, gam, gam}, {gam, bet, gam}, {gam, gam, bet}},
({{-delt}, {-delt}, {-pie}} + {{0}, {0}, {gam6o12}})]
U41 = Take[U4, {1, 1}]
U42 = Take[U4, {2, 2}]
U43 = Take[U4, {3, 3}]
gam1o7 = (1/U41)*((gam) + U41*sig + U42*(alph) + U43*(alph))
z9 = (1/gam3o9)*(U43*gam3o9 - bet - U41*alph - U43*alph - U43*sig)
gam8o9 = (-1/z9)*(gam + U41*alph + U42*alph + U43*alph)
gam7o9 = (-1/z9)*(gam + U41*sig + U42*alph +
U43*alph)
gam1o7=Det[gam1o7]
gam8o9=Det[gam8o9]
gam7o9=Det[gam7o9]

U5 = LinearSolve[{{bet, gam, gam}, {gam, bet, gam}, {gam, gam, bet}},
({{-delt}, {-pie}, {-delt}} + {{0}, {gam5o11}, {0}})]
U51 = Take[U5, {1, 1}]
U52 = Take[U5, {2, 2}]
U53 = Take[U5, {3, 3}]
z8 = (1/gam2o8)*(U52*gam2o8 - bet - U51*alph - U52*sig - U52*alph)
gam8o11 = (-1/z8)*(gam5o11)
gam7o8 = (-1/z8)*(gam + U41*sig + U42*alph +
U43*alph)
```

```
gam8o11=Det[gam8o11]
gam7o8=Det[gam7o8]

U6 = LinearSolve[{{bet, gam, gam}, {gam, bet, gam}, {gam, gam, bet}},
({{-sig}, {-alph}, {-alph}} + {{gam1o7}, {0}, {0}})]
U61 = Take[U6, {1, 1}]
U62 = Take[U6, {2, 2}]
U63 = Take[U6, {3, 3}]
z10 = (1/t)*(U61*t - bet - U61*pie - U62*delt - U63*delt)
gam7o10 = (-1/z10)*(gam1o7)
gam10o11 = (-1/z10)*(delt + U61*delt + U62*pie + U63*delt)
gam7o10=Det[gam7o10]
gam7o10=Det[gam7o10]
```

In this program, alph, bet, gam, delt, sig, and pie represent the equations derived from taking the response matrix for a layered 2 ring, 3 ray network. However, the output of this program is too long to print here. These outputs are then used as entries in the Kirchoff matrix, $K = X - (Y \cdot Z^{-1} \cdot Y')$ where

$$X = \begin{bmatrix} \gamma_{1,7} & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{2,8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{3,9} & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{4,10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{5,11} & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{6,12} \end{bmatrix}$$

$$Y = \begin{bmatrix} -\gamma_{1,7} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma_{2,8} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma_{3,9} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_{4,10} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma_{5,11} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma_{6,12} \end{bmatrix}$$

$$Z = \begin{bmatrix} \gamma_{1,7}+\gamma_{7,8}+\gamma_{7,9}+\gamma_{7,10} & -\gamma_{7,8} & -\gamma_{7,9} & \cdots \\ -\gamma_{7,8} & \gamma_{2,8}+\gamma_{7,8}+\gamma_{8,9}+\gamma_{8,11} & -\gamma_{8,9} & \cdots \\ -\gamma_{7,9} & -\gamma_{8,9} & \gamma_{3,9}+\gamma_{7,9}+\gamma_{8,9}+\gamma_{9,12} & \cdots \\ -\gamma_{7,10} & 0 & 0 & \cdots \\ 0 & -\gamma_{8,11} & 0 & \cdots \\ 0 & 0 & -\gamma_{9,12} & \cdots \end{bmatrix}$$

$$\begin{matrix} -\gamma_{7,10} & 0 & 0 \\ 0 & -\gamma_{8,11} & 0 \\ 0 & 0 & -\gamma_{9,12} \\ \gamma_{4,10}+\gamma_{7,10}+\gamma_{10,11}+\gamma_{10,12} & -\gamma_{10,11} & -\gamma_{10,12} \\ -\gamma_{10,11} & \gamma_{5,11}+\gamma_{8,11}+\gamma_{10,11}+\gamma_{11,12} & -\gamma_{11,12} \\ -\gamma_{10,12} & -\gamma_{11,12} & \gamma_{6,12}+\gamma_{9,12}+\gamma_{10,12}+\gamma_{11,12} \end{matrix}\Bigg]$$

which we then use to compute the response matrix. Because this response matrix is identical to the one we started with, it proves that $\gamma \rightarrow \Lambda$ is infinite to one, as there are infinitely many values of t which will produce identical response matrices.

In order to simplify the equations, we substitute $2alph + bet + 2gam$ for sig, and $2delt + bet + 2gam$ for pie. This reduces our final equations down to four variables, which are easier to solve in order to prove the response matrices are identical. In order to do so, we use the values of these gammas in our Kirchoff matrix, and compute a new Λ matrix. This new Λ matrix is the same as the original Λ matrix. Thus we can see that the mapping is infinite $\rightarrow$ one proving the conjecture.

Although by simplifying the results that are generated by Mathematica, we get that all values of t result in the same response matrix, this is not quite the case. For a finite number of values of t, the denominator of one of the four equations will be zero, thus creating a discontinuity. However, for all other real values of t, the response matrix is identical, thus proving that no layered network of 3 rays and 2 rings can be recovered. Some of these values of t will result in negative conductivities, however, which are physically impossible, however, mathematically, these are perfectly valid, symbolizing a resistor where current flows from the lower potential towards the higher potential.

1.1 Unlayered Networks

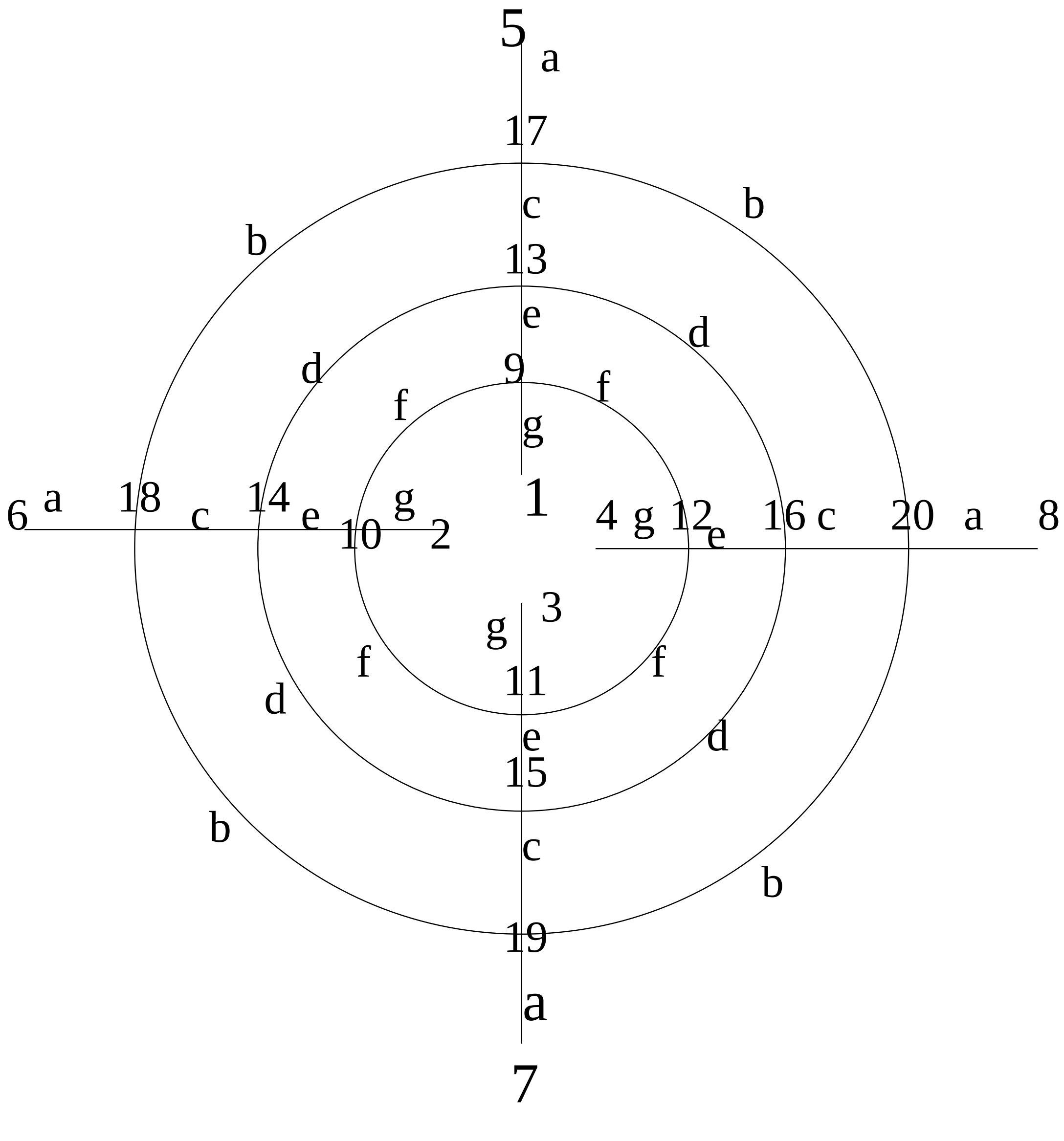
We can also look at the unlayered case. This is much more involved, requiring 21 variables, as opposed to the 4 needed in the layered case. This so far cannot be computed symbolically by Mathematica because each successive layer lengthens the equations, and they become too long to manipulate.

2 Recovery of a Layered 4 ray, 3 ring network

The other logical extension is the 4 ray, 3 ring layered graph. This has 8 boundary nodes and 28 conductors, so there is some hope of recovery, as $8 \cdot 7/2 = 28$. So, the next step is to write a program similar to the one for the 3 ray, 2 ring recovery to test this case. First, we generate the Λ matrix for the layered case:

$$\Lambda = \begin{bmatrix} \Sigma & \alpha & \beta & \alpha & \gamma & \delta & \epsilon & \delta \\ \alpha & \Sigma & \alpha & \beta & \delta & \gamma & \delta & \epsilon \\ \beta & \alpha & \Sigma & \alpha & \epsilon & \delta & \gamma & \delta \\ \alpha & \beta & \alpha & \Sigma & \delta & \epsilon & \delta & \gamma \\ \gamma & \delta & \epsilon & \delta & \Pi & \zeta & \eta & \zeta \\ \delta & \gamma & \delta & \epsilon & \zeta & \Pi & \zeta & \eta \\ \epsilon & \delta & \gamma & \delta & \eta & \zeta & \Pi & \zeta \\ \delta & \epsilon & \delta & \gamma & \zeta & \eta & \zeta & \Pi \end{bmatrix}$$

Next we run the following Mathematica program, also derived from Matlab code in [1]


5
a
17
c
b
b
13
e
d
d
9
f
f
g
6 a 18 c 14 e 10 g 2
1
4 g 12 e 16 c 20 a 8
3
g
f
f
11
d
d
e
15
b
c
b
19
a
7

```
U1=LinearSolve[{{gam,delt,eps,delt},{delt,gam,delt,eps},{eps,delt,gam,delt},
{delt,eps,delt,gam}},({{-pie},{-zet},{-eta},{-zet}}+{{a},{0},{0},{0}})]
U11=Simplify[Det[Take[U1,{1,1}]]
U21=Simplify[Det[Take[U1,{2,2}]]
U31=Simplify[Det[Take[U1,{3,3}]]
U41=Simplify[Det[Take[U1,{4,4}]]
U51=1

Z=({{sig,alph,bet,alph,gam,delt,eps,delt},{alph,sig,alph,bet,delt,gam,delt,eps},
{bet,alph,sig,alph,eps,delt,gam,delt},{alph,bet,alph,sig,delt,eps,delt,gam},
{gam,delt,eps,delt,pie,zet,eta,zet},{delt,gam,delt,eps,zet,pie,zet,eta},
{eps,delt,gam,delt,eta,zet,pie,zet},{delt,eps,delt,gam,zet,eta,zet,pie}}.
{{U11},{U21},{U31},{U41},{U51},{0},{0},{0}})
g=(1/U31)*(Det[Take[Z,{3,3}]])

U101=Simplify[(-1/g)*(-U21*g+(Det[Take[Z,{2,2}]]))
U121=Simplify[(-1/g)*(-U41*g+(Det[Take[Z,{4,4}]]))
U91=Simplify[(-1/g)*(-U11*g+(Det[Take[Z,{1,1}]]))
f=(U31*g)/(-U101-U121)
e=(-1/U121)*(f*(U121*2-U91)+g*(U121-U41))

U131=(1/e)*(U91*e+f*(2*U91-U101-U121)+g*(U91-U11))
d=(-U121*e)/U131
c=(-1/U131)*(2*U131*d+e*(U131-U91))

U2=LinearSolve[{{gam,delt,eps,delt},{delt,gam,delt,eps},{eps,delt,gam,delt},
{delt,eps,delt,gam}},({{-sig},{-alph},{-bet},{-alph}}+{{g},{0},{0},{0}})]
U52=Simplify[Det[Take[U2,{1,1}]]
U62=Simplify[Det[Take[U2,{2,2}]]
U72=Simplify[Det[Take[U2,{3,3}]]
U82=Simplify[Det[Take[U2,{4,4}]]
Z2=({{sig,alph,bet,alph,gam,delt,eps,delt},{alph,sig,alph,bet,delt,gam,delt,eps},
{bet,alph,sig,alph,eps,delt,gam,delt},{alph,bet,alph,sig,delt,eps,delt,gam},
{gam,delt,eps,delt,pie,zet,eta,zet},{delt,gam,delt,eps,zet,pie,zet,eta},
{eps,delt,gam,delt,eta,zet,pie,zet},{delt,eps,delt,gam,zet,eta,zet,pie}}.
{{1},{0},{0},{0},{U52},{U62},{U72},{U82}})

U182=Simplify[(-1/a)*(-U62*a+Take[Z2,{6,6}])]
U202=Simplify[(-1/a)*(-U82*a+Take[Z2,{8,8}])]
b=(a*U72)/(-U182-U202)

b=Det[Simplify[b]]
c=Det[Simplify[c]]
d=Det[Simplify[d]]
e=Det[Simplify[e]]
f=Det[Simplify[f]]
```

```
g=Det[Simplify[g]]
```

This gives us parametric equations for the conductors with a as the independent variable. By using these in our Kirchoff matrix, we can solve for the new Λ matrix, which should be the same as the original. This will thus prove that the mapping of $\gamma \to \Lambda$ is infinite $\to$ one, and thus that the network is also unrecoverable.

3 Semi-Conductor Networks

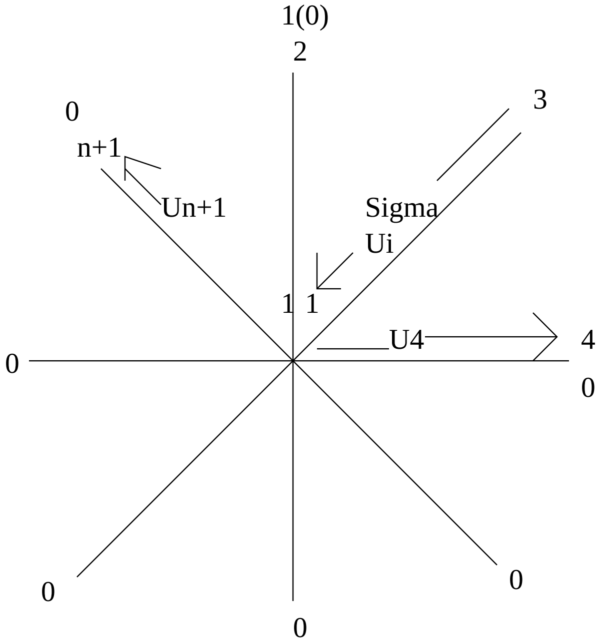
3.1 Recovery of Snowflake Vertex Conductivity Networks

It is possible to define a semi-conductor system for a network, as oppossed to the simple edge conductors we have considered so far. For these networks, we can set conductivities on the nodes, as oppossed to the edges. We then define the current flow between two nodes to be flowing from the node with higher potential to the node with lower potential, with the current being equal to the difference between these two potentials multiplied by the conductivity of the node with lower potential. This network precludes a response matrix, as it is non-linear, however, we will assume that the currents and voltages of the boundary nodes are measurable and alterable. The easiest such case is to have n boundary nodes, each of which is connected to the same, single interior node. Label the interior node 1, and the boundary nodes $2, 3, \ldots, n, n+1$. Set the voltage at node 2 to be 1, with current 0, and set the voltages at nodes 4 through $n+1$ to be 0. The voltage at node 1 will be 1, because this is the only such potential that will give node 2 a current of 0. This means that the currents flowing out towards 4 through $n+1$ will be equal to each of these node's conductivities, as the potential at node 1 is greater than 0, and the difference is equal to 1, so they are equal to 1 times the boundary node's conductivity. The current flowing in from node 3 must thus be equal to the sum of the conductivities of nodes 4 through $n+1$, so the voltage at node 3 must be equal to 1 plus the sum of those conductivities divided by the conductivity of node 1. As we assume that the voltages of the boundary nodes is measurable, and we know the sum of the conductivities of 4 through $n+1$ from above, we can determine the conductivity of node 1. By rotating the network, we can find the conductances of nodes 2 and 3, as well, thus recovering the entire network. This proves that it is possible to recover networks with very few boundary nodes in the vertex case, with more variables than the $n(n-1)/2$ limit found in the edge conductivity case.

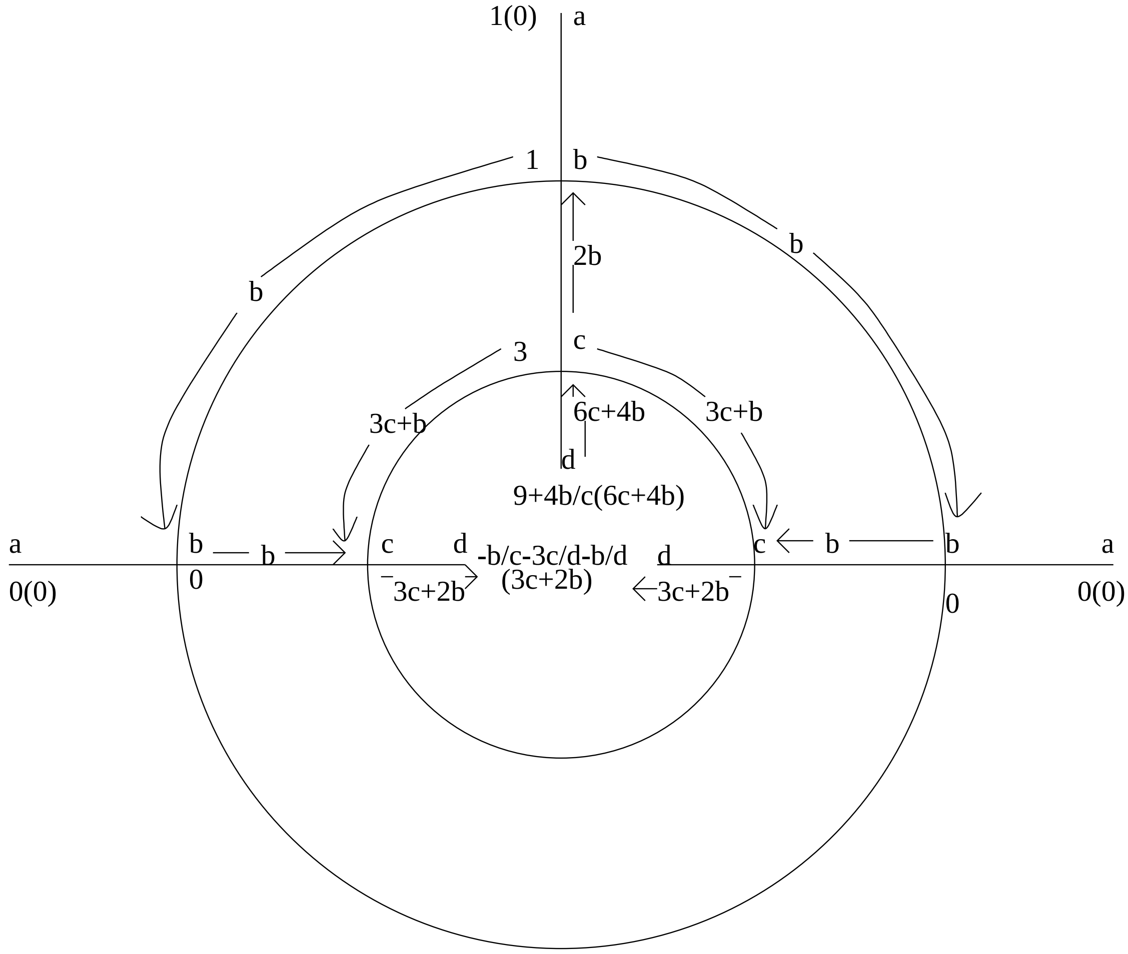
3.2 Circular Annular Semi-Conductor Networks

We can also construct such a network in the case of circular annular graphs. As usual, we will begin with the 3 ray, 2 ring case.

This case is recoverable in the layered case. Set the potential at node 1 to be 1, with current 0, and nodes 2 and 3 to have current and potential 0. This



makes the potential at node 7 be equal to 1, and the potentials at node 8 and 9 be 0. Thus current is flowing from node 7 to node 8, with a current equal to $(1-0)*b = b$. Similarly, the current flow from 7 to 9 is also b. As we know the current flow from node 1 to 7 is 0, the flow from 10 into 7 must be $2b$. Thus, $(U10-1)*b = 2b$, so the potential of node 10 is 3. The flow between nodes 8 and 9 is zero, as they have the same potential, and we know that the current flow from 2 to 8 is 0, so there must be a current b flowing out of node 8. Thus, $(0-U11)*c = b$, so the potential at 11 must be $-b/c$, and similarly, the potential at node 12 must also be $-b/c$. Now we can determine that the flow from node 10 to node 11 is equal to $(3-(-b/c))*c = 3c+b$, so the current flow from 10 to 12 is also equal to $3b+c$. Thus we have a total current flow out of node 10 equal to $(3c+b)+(3c+b)+(2b) = 6c+4b$. Thus the current flow from node 4 to node 10 must be equal to $6c+4b$. Thus, $(U4-3)*c = 6c+4b$, so Node 4 will now have a potential of $9+4b/c$, with current $6c+4b$. We also have a total current of $3c+b+b = 3c+2b$ flowing into node 11, so a current of $3c+2b$ must flow from node 11 to node 5, so $((-b/c)-U5)*d = 3c+2b$, and a similar argument holds for nodes 12 and 6. Thus, nodes 5 and 6 will have potentials of $-b/c-3c/d-b/d$. Given these three equations, b, c, and d can easily be recovered, and with these recovered, we can set nodes 1, 2, and 3 to have potential 0, with 4, 5, and 6 being potential 1. The current at 1, 2 and 3 will be abc, and with b and c known, a is easily obtained. Thus, the semi-conductor case is quite different from the edge conductivity case, as it is recoverable. This appears to be due to the fact the network is non-linear, so



that we can more effectively isolate conductances.

Similarly, we can have an algorithm for recovery of a 4 ray, 3 ring layered network using vertex conductivity. By using the graphs shown, we can make the recovery in the same way: by setting all the boundary nodes on the exterior to be currents 0, with one having a potential of 1 and the others having potential 0. We can then trace current flows inwards,and get a large number of equations. In this case, however, we must also reverse this procedure, and start from the inner ring of boundary nodes in order to solve, as well as the cases of setting all of one ring to have a potential of 1, with the other ring having potential 0. Let $\pi = \beta - \theta = -8b + 8d$, $\rho = 5\delta - 3\theta = -14b - 6c - 30d$, $\sigma = 4\kappa - 3\lambda = -27b - 3c + 5d$, $\tau = 5\rho + 14\sigma = -528b - 72c$, so $c = (528b - \tau)/(72)$, making $\iota = (528b - \tau) + (\beta - \theta)/8b + (\beta - \theta)/(8 * (528b - \tau))$, so we can recover b, then use this and τ to recover c, b and π to recover d, and finally, b, c, and d, along with ν and ξ to recover a and e.

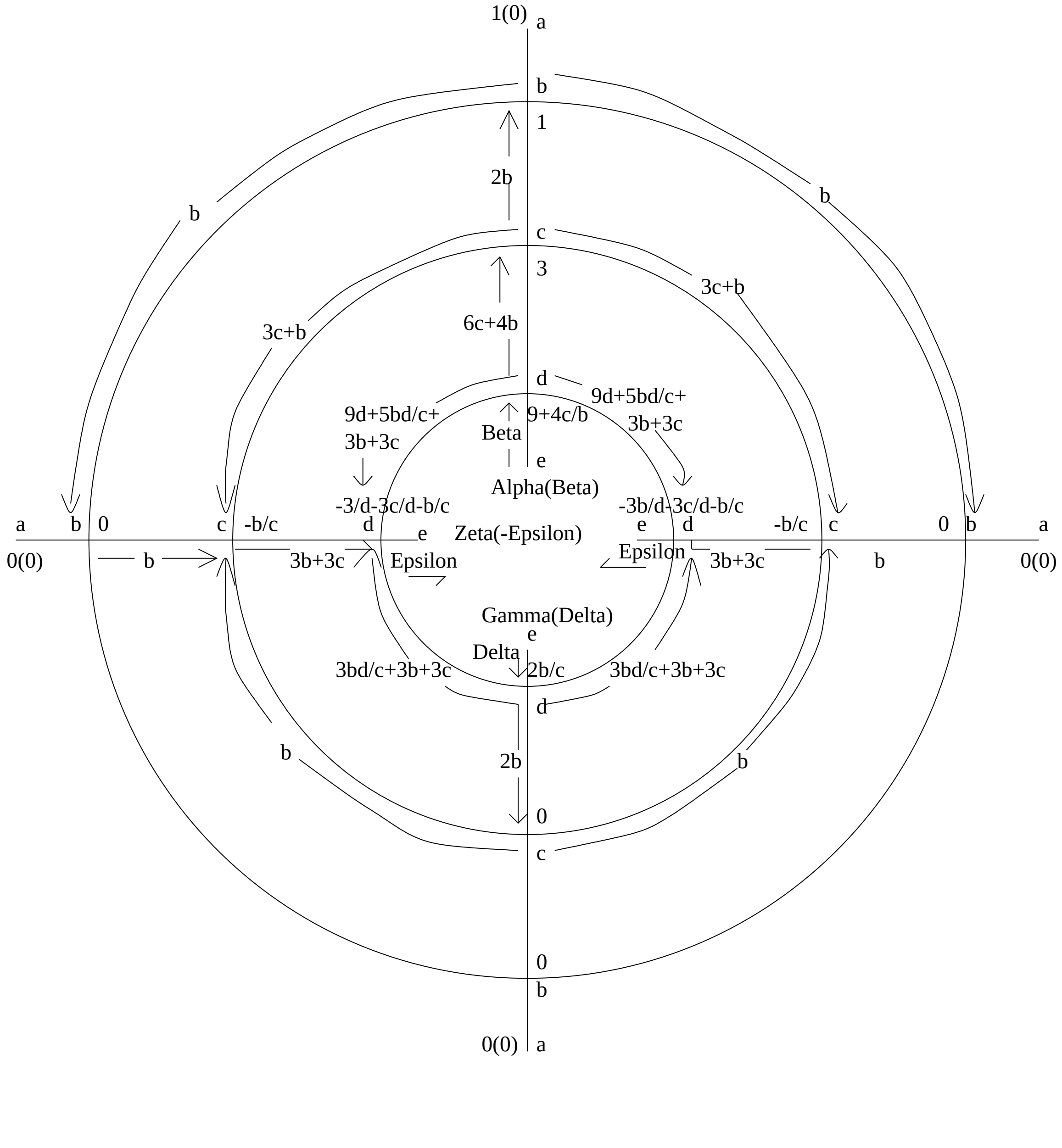
$\alpha = 27 + 4c/b + 10b/c + 10b/d + 12c$
$\beta = 18d + 10(b * d/c) + 10b + 12c$
$\gamma = 8b/c + 8b/d + 6c/d$
$\delta = 6(b * d/c) + 8b + 6c$
$\epsilon = 9d + 8(b * d/c) + 9b + 9c$
$\zeta = -9d/e - 8((b * d)/(c * e)) - 9b/e - 9c/e - 3b/d - 3c/d - b/c$
$\eta = 27 + 4c/d + 10d/c + 10d/b + 12c$
$\theta = 18b + 10(d * b/c) + 10d + 12c$
$\iota = 8d/c + 8d/b + 6c/b$
$\kappa = 6(d * b/c) + 8d + 6c$
$\lambda = 9b + 8(d * b/c) + 9d + 9c$
$\mu = -9b/a - 8((d * b)/(c * a)) - 9d/a - 3d/b - 3c/b - d/c$
$\nu = b * c * d * e$
$\xi = a * b * c * d$

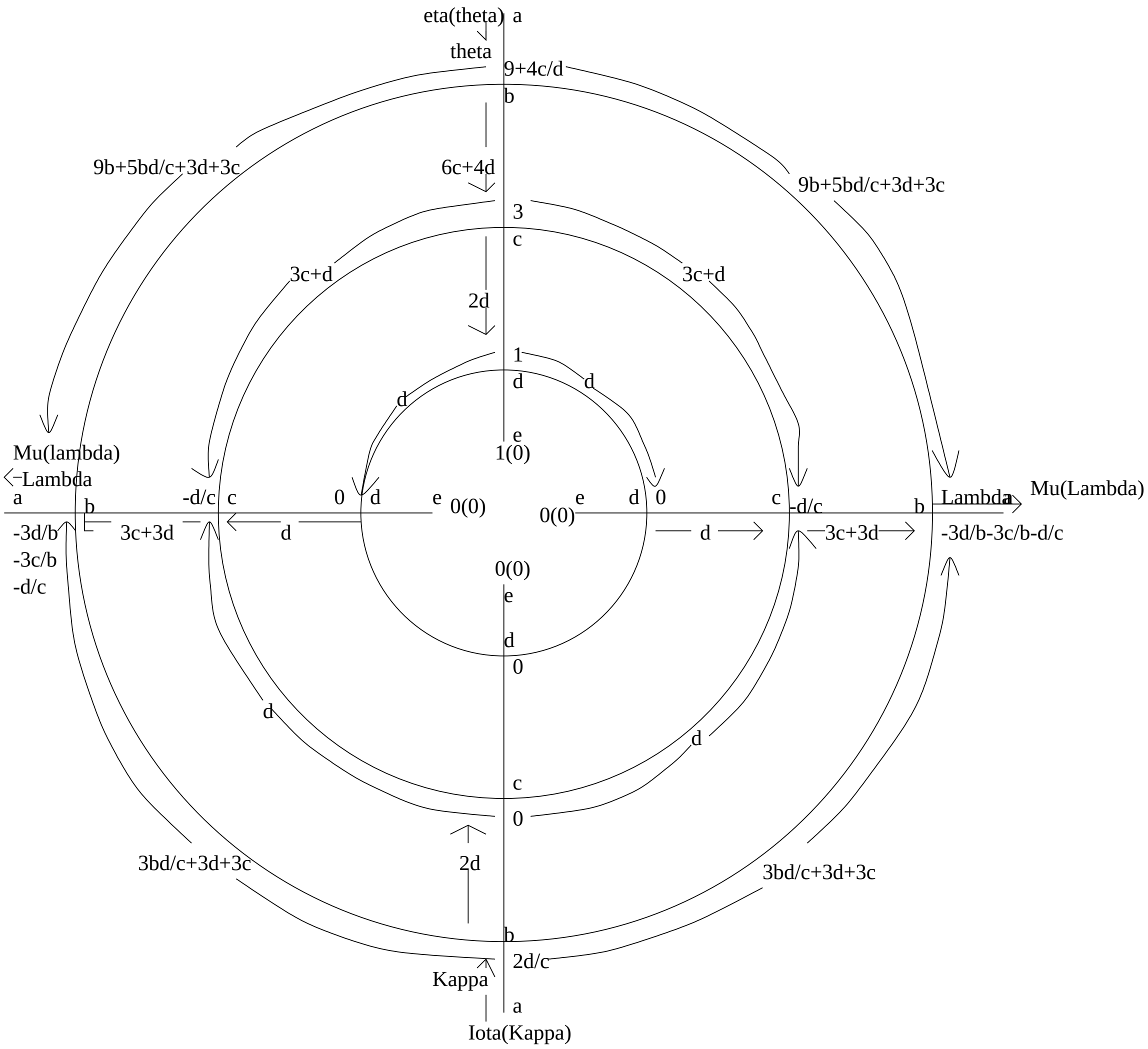
3.3 Conjecture

It is possible to recover any layered vertex conductivity network with n rays, and n rings. This would be done in a similar method to the above problems, however equations would be combined to a certain extent. However, there will still be more distinct equations than there are variables. For example, with 2 rings and 2 rays, we get equations that are $6c+4b$, $6c+4b$, $9+6b/c$, $-2b/c-(6c+6b)/d$, $a*b*c$, and $b*c*d$, which is an easily solvable system of equations.

References

[1] E. Esser: Recovery of Circular Annular Networks, University of Washington (2000)


1(0)
a
b
1
2b
c
3
6c+4b
d
9+4c/b
Beta
e
Alpha(Beta)
b
b
3c+b
3c+b
9d+5bd/c+
3b+3c
9d+5bd/c+
3b+3c
-3/d-3c/d-b/c
-3b/d-3c/d-b/c
a
b
0
c
-b/c
d
e
Zeta(-Epsilon)
e
d
-b/c
c
0
b
a
0(0)
b
3b+3c
Epsilon
Epsilon
3b+3c
b
0(0)
Gamma(Delta)
e
Delta
2b/c
3bd/c+3b+3c
3bd/c+3b+3c
d
b
2b
b
0
c
0
b
0(0)
a


eta(theta)
a
theta
9+4c/d
b
9b+5bd/c+3d+3c
6c+4d
9b+5bd/c+3d+3c
3
c
3c+d
3c+d
2d
1
d
d
d
e
1(0)
Mu(lambda)
Lambda
a
b
-d/c
c
0
d
e
0(0)
0(0)
e
d
0
c
-d/c
b
Lambda
Mu(Lambda)
-3d/b
-3c/b
-d/c
3c+3d
d
d
3c+3d
-3d/b-3c/b-d/c
0(0)
e
d
0
d
d
c
0
3bd/c+3d+3c
2d
3bd/c+3d+3c
b
2d/c
Kappa
a
Iota(Kappa)