three nodes in one circle

never has only one pairing
2-nodes in 1-circle, 1 in 2 other circles. All inner circles.
a nodes in 3 different circles

det(1,2,4,3)

from inner A

det(1,7,12,8)

det(1,7,11,10)

using 3 inner circles

using 3 circles, middle pocket circle
One node in each of four circles

necessarily has only one pairing.
all 4 nodes on "inner triangle"

\begin{align*}
\text{det}(1, 2, 3, 4) \\
\text{det}(1, 4, 2, 3) \\
\text{det}(1, 2, 6, 3) \\
\text{det}(1, 6, 2, 4)
\end{align*}

always has one vanishing!!
3 nodes in one circle, 2 in one, 1 in one.

\[ \text{det}(7, 2, 3, 8) > 0 \]
\[ \text{det}(1, 2, 7, 3, 6, 9) > 0 \]
\[ \text{det}(1, 2, 7, 3, 6, 9) < 0 \]
\[ \text{det}(5, 6, 7, 13, 9) > 0 \]
\[ \text{det}(1, 6, 7, 3, 4, 8) < 0 \]
\[ \text{det}(1, 7, 3, 4, 8) < 0 \]
\[ \text{det}(1, 4, 7, 2, 5, 3) < 0 \]
\[ \text{det}(1, 4, 7, 2, 5, 3) < 0 \]
\[ \text{det}(1, 2, 6, 7, 1, 9) < 0 \]

All can make only one pairing!!
2 nodes in one circle, 2 in 2 others

\[
\det(1,2,5;3,8,9) > 0
\]

\[
\det(1,2,4;7,8,9) < 0
\]

\[
\det(1,5,7;2,4,8) < 0
\]

\[
\det(3,4,9;1,5,7) > 0
\]

\[
\det(1,3,8;4,7,9) > 0
\]

\[
\det(1,2,7;3,5,9) > 0
\]

\[
\det(1,2,5;3,8,7) < 0
\]
Nodes in two circles, 1 in two circles

\[
\det(1,5,7;2,3,10) < 0 \\
\det(2,3,4;1,7,10) > 0 \\
\det(1,2,3;7,9,10) > 0
\]

\[
\det(1,5,7;2,3,11) < 0 \\
\det(2,3,11;1,4,7) < 0 \\
\det(1,2,3;7,9,11) > 0
\]

\[
\det(1,2,7;3,5,12) > 0 \\
\det(1,2,7;3,4,12) > 0 \\
\det(1,2,7;3,9,12) > 0
\]

\[
\det(1,2,7;5,8,10) < 0 \\
\det(2,4,11;5,7,10) < 0 \\
\det(1,2,9,7;5,10) < 0
\]
3 nodes in one circle, 8 nodes in another

\[ \det \begin{pmatrix} 1 & 2 & 7 \\ 3 & 4 & 8 \\ 5 & 6 & 9 \end{pmatrix} < 0 \]

always only one passing!
3 nodes in 1 circle, 1 in 3 others

\[
\begin{align*}
\det(1,6,7;2,5,10) &< 0 \\
\det(1,6,7;2,5,11) &< 0 \\
\det(1,6,7;2,4,10) &< 0 \\
\det(1,6,7;2,4,11) &> 0 \\
\det(1,6,7;4,5,12) &> 0 \\
\det(1,4,7;2,9,10) &< 0 \\
\det(2,6,7;2,7,9,11) &> 0 \\
\det(1,6,7;3,9,12) &< 0 \\
\det(1,6,7;3,4,11) &< 0 \\
\det(1,7,10;3,6,9) &< 0 \\
\det(1,7,10;3,6,9) &< 0 \\
\det(1,6,7;8,9,10) &< 0 \\
\det(1,6,7;8,9,11) &< 0
\end{align*}
\]
\[ \det(1, 4, 6, 7) > 0 \]
\[ \det(1, 2, 3, 6; 4, 5, 7, 8) > 0 \]
\[ \det(1, 2, 3, 7; 4, 5, 6, 8) > 0 \]
\[ \det(1, 2, 4, 8; 3, 5, 6, 7) > 0 \]
\[ \det(1, 2, 5, 7; 3, 4, 6, 8) > 0 \]
\[ \det(1, 3, 4, 8; 2, 5, 6, 7) > 0 \]
\[ \det(1, 4, 5, 6; 2, 3, 7, 8) < 0 \]
\[ \det(1, 5, 6, 8; 2, 3, 4, 7) > 0 \]
\[ \det(1, 6, 7, 8; 2, 3, 4, 6) < 0 \]
\[ \det(1, 4, 6, 9; 2, 3, 7, 8) > 0 \]
\[ \det(1, 2, 3, 6, 7, 8, 9) > 0 \]
\[ \det(1, 2, 3, 7, 4, 6, 8, 9) > 0 \]
\[ \det(1, 2, 4, 5, 6, 7, 8, 9) > 0 \]
\[ \det(1, 2, 4, 7, 3, 8, 6, 9) < 0 \]
\[ \det(1, 2, 7, 8, 3, 4, 6, 9) < 0 \]
\[ \det(1, 2, 8, 9, 3, 4, 6, 7) < 0 \]
\[ \det(1, 4, 5, 6, 7, 8, 9) > 0 \]
\[ \det(1, 4, 6, 7, 3, 8, 9) < 0 \]
\[ \det(1, 4, 7, 8, 2, 3, 6, 9) > 0 \]
\[ \det(1, 7, 8, 9, 2, 3, 4, 6) > 0 \]
3 in 2 circles, 1 in two others

\[
\det (1, 3, 7, 6; 2, 8, 9, 5) > 0
\]
*See next page*

\[
\det (1, 3, 6, 7; 2, 9, 9, 10) > 0
\]
\[
\det (1, 6, 7, 10; 2, 3, 4, 8) > 0
\]

\[
\det (1, 6, 7, 8; 2, 3, 9, 12) > 0
\]

\[
\det (1, 6, 7, 9; 2, 3, 5, 12) > 0
\]
4.4 2 in four circles

\[
\begin{align*}
\text{det}(1,4,6,12; 2,3,5,11) & > 0 \\
\text{det}(1,4,11,12; 2,5,9,11) & > 0 \\
\text{det}(1,5,7,11; 4,5,11) & < 0 \\
\text{det}(3,4,7,11; 4,5,11) & < 0 \\
\text{det}(2,6,9,12; 3,4,7,11) & < 0 \\
\text{det}(2,6,9,12; 4,6,9,11) & < 0 \\
\text{det}(1,6,9,12; 4,6,9,11) & < 0 \\
\text{det}(2,7,9,11; 3,4,7,12) & > 0 \\
\text{det}(2,7,9,11; 4,6,9,11) & < 0 \\
\text{det}(2,7,9,11; 5,6,9,11) & > 0 \\
\text{det}(2,7,9,11; 3,5,6,11) & < 0 \\
\text{det}(3,4,7,9; 6,9,11) & > 0 \\
\text{det}(3,4,7,9; 5,6,9,11) & > 0 \\
\text{det}(3,4,7,9; 5,6,9,11) & > 0
\end{align*}
\]
3 in one circle, 2 in two circles, 1 in one circle ≤
S-5 3 in 3 circles, 1 in one circle

\[ \det(1, 2, 3, 4, 5, 6, 7, 8, 9, 10) > 0 \]
Yes in 2 circles; 7 in 2 circles

\[ \det (1,4,6,7,11; 2,3,5,8,10) > 0 \]

\[ \det (1,4,6,7,11; 2,3,5,8,12) > 0 \]

\[ \det (1,6,7,9,11; 2,3,5,8,10) > 0 \]

\[ \det (1,6,7,9,11; 2,3,5,8,12) > 0 \]

\[ \det (1,4,7,9,12; 2,3,5,7,10) > 0 \]
all boundary nodes

\[ \det \left( 1, 0, 6, 7, 9, 11, 2, 3, 5, 8, 10, 12 \right) > 0 \]