**Appendix A: Calculations of Algebraic Impossibility**

To keep track of the values of determinant pairs, we will use the notation from the proof of the theorem in Section 3.3. The values of the determinants will be denoted using $a, b, c, d, e, f, g, h, j, k$ from the general Kirchhoff matrix in Section 2.2, as in Table 2.

It is worth noting that renumbering boundary nodes in either circular or reverse circular order does not change how the determinanant values interact, because the beginning node and direction of a numbering is arbitrary. This means, for example, that once we have shown

$$\ominus \oplus + + +$$

to be an algebraically impossible combination of determinants, we also will know that

$$+ \ominus \oplus + + \text{ and } + + \ominus \oplus + \text{ and } + + + \ominus \oplus \text{ and } \oplus + + + \ominus$$

are impossible combinations, as are

$$+{+}{+}\oplus\ominus \text{ and } {+}{+}\oplus\ominus{+} \text{ and } {+}\oplus\ominus{+}{+} \text{ and } \oplus\ominus{+}{+}{+} \text{ and } \ominus{+}{+}{+}\oplus.$$

Therefore, when one combination is found to be impossible, all its renumberings in circular and reverse circular order will also be impossible, based on the proof of the original impossibility.

In some cases, only a segment of the combination is needed to prove the impossibility. When this occurs, all other combinations of determinants which contain this segment in either circular or reverse circular order will also be impossible, based on the proof of the impossibility of the original segment.

We will refer to a reordering as *clean* if the new $\Lambda$ has two pairs of problematic determinants only if they are adjacent.

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|---|---|---|
| 1 | $\ominus \ominus \ominus \ominus \ominus$ | There is a recoverable **circular planar network** with this $\Lambda$. |
| 2 | $\ominus \ominus \ominus \ominus +$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 3 | $\ominus \ominus \ominus \ominus \oplus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |

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|---|---|---|
| 4 | $\ominus\ominus\ominus+\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 5 | $\ominus\ominus\ominus++$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 6 | $\ominus\ominus\ominus+\oplus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 7 | $\ominus\ominus\ominus\oplus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 8 | $\ominus\ominus\ominus\oplus+$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 9 | $\ominus\ominus\ominus\oplus\oplus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 10 | $\ominus\ominus+\ominus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 11 | $\ominus\ominus+\ominus+$ | **Impossible.** See Section 3.2. |
| 12 | $\ominus\ominus+\ominus\oplus$ | **Impossible.** See Section 3.2. |
| 13 | $\ominus\ominus++\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 14 | $\ominus\ominus+++$ | Reorder nodes **12543**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 15 | $\ominus\ominus++\oplus$ | Reorder nodes **12543**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 16 | $\ominus\ominus+\oplus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 17 | $\ominus\ominus+\oplus+$ | Reorder nodes **12543**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 18 | $\ominus\ominus+\oplus\oplus$ | Reorder nodes **12543**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 19 | $\ominus\ominus\oplus\ominus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |

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| 20 | $\ominus\ominus\oplus\ominus+$ | **Impossible.** See Section 3.2. |
| 21 | $\ominus\ominus\oplus\ominus\oplus$ | **Impossible.** See Section 3.2. |
| 22 | $\ominus\ominus\oplus+\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 23 | $\ominus\ominus\oplus++$ | Reorder nodes **12543**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 24 | $\ominus\ominus\oplus+\oplus$ | Reorder nodes **13254**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 25 | $\ominus\ominus\oplus\oplus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 26 | $\ominus\ominus\oplus\oplus+$ | Reorder nodes **12543**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 27 | $\ominus\ominus\oplus\oplus\oplus$ | For all intents and purposes, this combination is **impossible.** The important inequalities are: <br> $cj - dh \leq 0 \quad \text{(Index 2)}$ <br> $aj \geq de \quad \text{(Index 4)}$ <br> $ce \geq ah \quad \text{(Index 5)}$ <br> Now, we can treat the last as a strict inequality, because if both determinants of index 3 are equal, and both determinants of index 5 are equal, this combination could be treated as #17. Symmetry says that making the determinants of index 5 strictly not equal has the same effect as restricting the determinants of index 3. So $ce > ah$. Then $c \neq 0$ so $e > \frac{ah}{c}$ and <br> $0 \leq aj - de < aj - d\dfrac{ah}{c} = a(cj - dh)$ <br> but $cj - dh \leq 0$ and $a \geq 0$ and this is a contradiction. |

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| 28 | $\ominus + \ominus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 29 | $\ominus + \ominus \ominus +$ | **Impossible.** See Section 3.2. |
| 30 | $\ominus + \ominus \ominus \oplus$ | **Impossible.** See Section 3.2. |
| 31 | $\ominus + \ominus + \ominus$ | **Impossible.** See Section 3.2. |
| 32 | $\ominus + \ominus + +$ | **Impossible.** It is only necessary to consider determinant pairs 1, 2 and 5. $$fj - gh \leq 0 \text{ and } fj - ek \leq 0 \text{ (Index 1)}$$ $$cj - bk > 0 \text{ and } dh > bk \text{ (Index 4)}$$ $$bf - ce > 0 \text{ and } ah > ce \text{ (Index 5)}$$ Now, since $bf - ce > 0$ and $cj - bk > 0$ we know that $b, f, c, j \neq 0$. This means that $fj > 0$ so $e, k \neq 0$ to make $fj - ek \leq 0$. So from determinant pair 5, we know that $f > \frac{ce}{b}$. Substitution tells us that $$0 \geq fj - ek > \frac{ce}{b}j - ek$$ . Since $e \neq 0$ this means that $$0 > cj - bk$$ which is a contradiction. Therefore, the segment $+ \oplus +$ can never occur. |
| 33 | $\ominus + \ominus + \oplus$ | **Impossible.** See #32. |
| 34 | $\ominus + \ominus \oplus \ominus$ | **Impossible.** See Section 3.2. |
| 35 | $\ominus + \ominus \oplus +$ | **Impossible.** See #32. |

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| 36 | $\ominus + \ominus \oplus \oplus$ | **Impossible.** The important inequalities are: <br><br> $fj - ek \leq)$ (Index 1) <br> $cj - bk > 0$ and $dh > bk$ (Index 2) <br> $cg - ak \leq 0$ (Index 3) <br> $aj \geq de$ and $bg - de > 0$ ((Index 4) <br> $bf - ah > 0$ (Index 5) <br><br> Determinants 2, 4 and 5 tell us that $c, j, g, b, f \neq 0$, and therefore the determinants 1 and 3 tell us that $e, k, a \neq 0$. So we can say that $j \geq \frac{de}{a}$ and $j \leq \frac{ek}{f}$. This means that $\frac{de}{a} \leq \frac{ek}{f}$ so $d \leq \frac{ak}{f}$. Substitution gives <br><br> $$\frac{ak}{f}h > dh > bk$$ <br> or $ah > bf$. But $ah < bf$ so this is a contradiction. |
| 37 | $\ominus + + \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 38 | $\ominus + + \ominus +$ | **Impossible.** See #32. |
| 39 | $\ominus + + \ominus \oplus$ | **Impossible.** The important inequalities are: <br><br> $fj - gh \leq 0$ and $fj - ek \leq 0$ (Index 1) <br> $cj - bk > 0$ and $dh \geq bk$ (Index 2) <br> $cg - df > 0$ and $ak \geq df$ (Index 3) <br> $bf - ah > 0$ (Index 5) <br><br> The first inequalities from determinant pairs 2,3 and 5 tell us that $c, j, g, b, f \neq 0$. This in turn tells us, using the determinants of index 1, that$h, k \neq 0$. Therefore, we can say that $b \leq \frac{dh}{k}$ and $b > \frac{ah}{f}$ so $\frac{ah}{f} < \frac{dh}{k}$. Since $h \neq 0$, $ak < df$ which is a contradiction. Therefore, the combination $\ominus + + \ ? \oplus$ is algebraically impossible. |

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| 40 | $\ominus + + + \ominus$ | Reorder nodes **14325**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 41 | $\ominus + + + +$ | **Impossible.** See #32. |
| 42 | $\ominus + + + \oplus$ | **Impossible.** See #39. |
| 43 | $\ominus + + \oplus \ominus$ | Reorder nodes **14325**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 44 | $\ominus + + \oplus +$ | **Impossible.** See #32. |
| 45 | $\ominus + + \oplus \oplus$ | **Impossible.** See #39. |
| 46 | $\ominus + \oplus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 47 | $\ominus + \oplus \ominus +$ | **Impossible.** See #32. |
| 48 | $\ominus + \oplus \ominus \oplus$ | **Impossible.** See #39. |
| 49 | $\ominus + \oplus + \ominus$ | Reorder nodes **14325**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 50 | $\ominus + \oplus + +$ | **Impossible.** See #32. |
| 51 | $\ominus + \oplus + \oplus$ | Reorder nodes **13254**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 52 | $\ominus + \oplus \oplus \ominus$ | Reorder nodes **14325**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 53 | $\ominus + \oplus \oplus +$ | **Impossible.** See #32. |
| 54 | $\ominus + \oplus \oplus \oplus$ | **Impossible.** The important inequalities are: $fj - gh \leq 0$ and $fj - ek \leq 0$ (Index 1)<br>$cj - bk > 0$ and $dh > bk$ (Index 2)<br>$df \geq ak$ (Index 3)<br>$aj \geq de$ and $bg - de > 0$ (Index 4)<br>$bf - ah > 0$ (Index 5) |

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| | | Determinants from pairs 2, 3, 4 and 5 tell us that $c, j, g, b, f \neq 0$ which tells us that in the inequalities of index 1, $g, h, e, k \neq 0$. This in turn tells us that since $bk > 0$, $d \neq 0$. Then $de > 0$ so $a \neq 0$. With this knowledge, we can say that $f \geq \frac{ak}{d}$. Substitution gives $$\frac{ak}{d}j - ek \leq fj - ek \leq 0$$ so $aj - de \leq 0$. But $aj - de \geq 0$ so $aj = de$. This means that $bg - aj > 0$ so $b > \frac{aj}{g}$ and $$\frac{aj}{g}f - ah > bf - ah > 0$$ or $fj - gh > 0$. But this is a contradiction. |
| 55 | $\ominus \oplus \ominus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 56 | $\ominus \oplus \ominus \ominus +$ | **Impossible.** See Section 3.2. |
| 57 | $\ominus \oplus \ominus \ominus \oplus$ | **Impossible.** See Section 3.2. |
| 58 | $\ominus \oplus \ominus + \ominus$ | **Impossible.** See Section 3.2. |
| 59 | $\ominus \oplus \ominus + +$ | **Impossible.** See #39. |
| 60 | $\ominus \oplus \ominus + \oplus$ | **Impossible.** See #39. |
| 61 | $\ominus \oplus \ominus \oplus \ominus$ | **Impossible.** See Section 3.2. |
| 62 | $\ominus \oplus \ominus \oplus +$ | **Impossible.** See #39. |
| 63 | $\ominus \oplus \ominus \oplus \oplus$ | **Impossible.** The important determinants are: $fj - gh \leq 0$ (Index 1)<br>$cj - dh > 0$ (Index 2)<br>$cg - df \leq 0$ and $cg - ak \leq 0$ (Index 3)<br>$bg - de > 0$ (Index 4)<br>$bf - ah > 0$ (Index 5) |

| | | |
|---|---|---|
| | | Determinants from index 2, 4 and 5 tell us that $c, j, b, g, f \neq 0$. Thus, determinants of index 3 tell us that $d, f, a, k \neq 0$, and $c \leq \frac{df}{g}$. Substitution gives $$\frac{df}{g}j - dh \geq cj - dh > 0$$ so $fj - gh > 0$ which is a contradiction. |
| 64 | $\ominus \oplus + \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 65 | $\ominus \oplus + \ominus +$ | **Impossible.** See #32. |
| 66 | $\ominus \oplus + \ominus \oplus$ | **Impossible.** See #39. |
| 67 | $\ominus \oplus + + \ominus$ | Reorder nodes **14325**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 68 | $\ominus \oplus + + +$ | **Impossible.** See #39. |
| 69 | $\ominus \oplus + + \oplus$ | Reorder nodes **13254**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 70 | $\ominus \oplus + \oplus \ominus$ | Reorder nodes **12534**; $\Lambda$ has two adjacent pairs of problematic determinants. All other re-orderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 71 | $\ominus \oplus + \oplus +$ | Reorder nodes **12543**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 72 | $\ominus \oplus + \oplus \oplus$ | Reorder nodes **12543**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 73 | $\ominus \oplus \oplus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 74 | $\ominus \oplus \oplus \ominus +$ | **Impossible.** See #36. |
| 75 | $\ominus \oplus \oplus \ominus \oplus$ | **Impossible.** See #63. |

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| 76 | $\ominus \oplus \oplus + \ominus$ | Reorder nodes **14325**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 77 | $\ominus \oplus \oplus + +$ | **Impossible.** See #39. |
| 78 | $\ominus \oplus \oplus + \oplus$ | Reorder nodes **14325**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 79 | $\ominus \oplus \oplus \oplus \ominus$ | **Impossible.** See #27. |
| 80 | $\ominus \oplus \oplus \oplus +$ | **Impossible.** See #54. |
| 81 | $\ominus \oplus \oplus \oplus \oplus$ | For all intents and purposes, this combination is **impossible.** The important inequalities are: <br> $fj - ek \leq 0$ (Index 1) <br> $cj - dh > 0$ (Index 2) <br> $df \geq ak$ (Index 3) <br> $aj \geq de$ (Index 4) <br> If both determinants of index 3 are equal, and both determinants of index 4 are equal, then this combination could be treated as #69. So let $df > ak$. Symmetry makes this the same as making the inequality of index 4 strict. Then $f \neq 0$ and determinant 2 tells us that $j \neq 0$. Since $fj > 0$, $e \neq 0$ as well. Thus we can say that $\frac{ak}{f} < d$ and $\frac{aj}{e} \geq d$ so $\frac{ak}{f} < \frac{aj}{e}$ Therefore, <br> $0 < \frac{aj}{e} - \frac{ak}{f} = \frac{a}{ef}(fj - ek)$ <br> but $fj - ek \leq 0$ so this is a contradiction. |

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|---|---|---|
| 82 | $+\ominus\ominus\ominus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 83 | $+\ominus\ominus\ominus+$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 84 | $+\ominus\ominus\ominus\oplus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 85 | $+\ominus\ominus+\ominus$ | **Impossible.** See Section 3.2. |
| 86 | $+\ominus\ominus++$ | Reorder nodes **13245**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 87 | $+\ominus\ominus+\oplus$ | Reorder nodes **13245**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 88 | $+\ominus\ominus\oplus\ominus$ | **Impossible.** See Section 3.2. |
| 89 | $+\ominus\ominus\oplus+$ | Reorder nodes **13245**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 90 | $+\ominus\ominus\oplus\oplus$ | Reorder nodes **13245**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 91 | $+\ominus+\ominus\ominus$ | **Impossible.** See Section 3.2. |
| 92 | $+\ominus+\ominus+$ | **Impossible.** See #32. |
| 93 | $+\ominus+\ominus\oplus$ | **Impossible.** See #32. |
| 94 | $+\ominus++\ominus$ | **Impossible.** See #32. |
| 95 | $+\ominus+++$ | **Impossible.** See #32. |
| 96 | $+\ominus++\oplus$ | **Impossible.** See #32. |
| 97 | $+\ominus+\oplus\ominus$ | **Impossible.** See #32. |
| 98 | $+\ominus+\oplus+$ | **Impossible.** See #32. |
| 99 | $+\ominus+\oplus\oplus$ | **Impossible.** See #32. |
| 100 | $+\ominus\oplus\ominus\ominus$ | **Impossible.** See Section 3.2. |
| 101 | $+\ominus\oplus\ominus+$ | **Impossible.** See #39. |

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| 102 | $+ \ominus \oplus \ominus \oplus$ | **Impossible.** See #39. |
| 103 | $+ \ominus \oplus + \ominus$ | **Impossible.** See #32. |
| 104 | $+ \ominus \oplus + +$ | **Impossible.** See #39. |
| 105 | $+ \ominus \oplus + \oplus$ | Reorder nodes **13245**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 106 | $+ \ominus \oplus \oplus \ominus$ | **Impossible.** See #36. |
| 107 | $+ \ominus \oplus \oplus +$ | **Impossible.** See #39. |
| 108 | $+ \ominus \oplus \oplus \oplus$ | **Impossible.** See #54. |
| 109 | $+ + \ominus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 110 | $+ + \ominus \ominus +$ | Reorder nodes **12435**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 111 | $+ + \ominus \ominus \oplus$ | Reorder nodes **12435**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 112 | $+ + \ominus + \ominus$ | **Impossible.** See #32. |
| 113 | $+ + \ominus + +$ | **Impossible.** See #32. |
| 114 | $+ + \ominus + \oplus$ | **Impossible.** See #32. |
| 115 | $+ + \ominus \oplus \ominus$ | **Impossible.** See #39. |
| 116 | $+ + \ominus \oplus +$ | **Impossible.** See #39. |
| 117 | $+ + \ominus \oplus \oplus$ | **Impossible.** See #39. |
| 118 | $+ + + \ominus \ominus$ | Reorder nodes **12354**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 119 | $+ + + \ominus +$ | **Impossible.** See #32. |
| 120 | $+ + + \ominus \oplus$ | **Impossible.** See #42. |
| 121 | $+ + + + \ominus$ | **Impossible.** See #32. |

| | | |
|---|---|---|
| 122 | $+ + + + +$ | **Impossible.** The important inequalities are:<br>$cj - bk > 0$ (Index 2)<br>$ak > df$ (Index 3)<br>$de > aj$ (Index 4)<br>$bf - ce > 0$ (Index 5)<br>Now, note that $c, j, a, k, d, e, b, f \neq 0$ from these inequalities, so we can say that $f < \frac{ak}{d}$ and substitution gives<br>$fj - ek < \frac{ak}{d} j - ek = \frac{k}{d}(aj - de) < 0$<br>since $de > aj$. So $fj - ek < 0$ and $f < \frac{ek}{j}$. Another substitution gives<br>$0 < bf - ce < b\frac{ek}{j} - ce = \frac{e}{j}(bk - cj) < 0$<br>because $cj - bk > 0$. But then $0 < bf - ce < 0$, which is a contradiction. |
| 123 | $+ + + + \oplus$ | **Impossible.** See the proof used as an example in the proof of the theorem in Section 3.3. Since the only determanant pairs used in the proof were pairs 2, 3 and 5, that proof essentially proved that any combination $? ++ ? \oplus$ is algebraically impossible. |
| 124 | $+ + + \oplus \ominus$ | **Impossible.** See #39. |
| 125 | $+ + + \oplus +$ | **Impossible.** See #123. |
| 126 | $+ + + \oplus \oplus$ | **Impossible.** See #123. |
| 127 | $+ + \oplus \ominus \ominus$ | Reorder nodes **12354**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 128 | $+ + \oplus \ominus +$ | **Impossible.** See #39. |
| 129 | $+ + \oplus \ominus \oplus$ | Reorder nodes **14235**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |

| | | |
|---|---|---|
| 130 | $++\oplus+\ominus$ | **Impossible.** See #32. |
| 131 | $++\oplus++$ | **Impossible.** See #123. |
| 132 | $++\oplus+\oplus$ | Reorder nodes **14235**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 133 | $++\oplus\oplus\ominus$ | **Impossible.** See #39. |
| 134 | $++\oplus\oplus+$ | **Impossible.** See #123. |
| 135 | $++\oplus\oplus\oplus$ | **Impossible.** See #123 |
| 136 | $+\oplus\ominus\ominus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 137 | $+\oplus\ominus\ominus+$ | Reorder nodes **12435**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 138 | $+\oplus\ominus\ominus\oplus$ | Reorder nodes **12453**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 139 | $+\oplus\ominus+\ominus$ | **Impossible.** See #32. |
| 140 | $+\oplus\ominus++$ | **Impossible.** See #39. |
| 141 | $+\oplus\ominus+\oplus$ | Reorder nodes **12453**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 142 | $+\oplus\ominus\oplus\ominus$ | **Impossible.** See #39. |
| 143 | $+\oplus\ominus\oplus+$ | Reorder nodes **12453**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 144 | $+\oplus\ominus\oplus\oplus$ | Reorder nodes **13245**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 145 | $+\oplus+\ominus\ominus$ | Reorder nodes **12354**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |

| | | |
|---|---|---|
| 146 | $+ \oplus + \ominus +$ | **Impossible.** See #32. |
| 147 | $+ \oplus + \ominus \oplus$ | Reorder nodes **12354**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 148 | $+ \oplus + + \ominus$ | **Impossible.** See #32. |
| 149 | $+ \oplus + + +$ | **Impossible.** See #123. |
| 150 | $+ \oplus + + \oplus$ | Reorder nodes **13254**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 151 | $+ \oplus + \oplus \ominus$ | Reorder nodes **12534**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 152 | $+ \oplus + \oplus +$ | Reorder nodes **12435**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 153 | $+ \oplus + \oplus \oplus$ | Reorder nodes **12354**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 154 | $+ \oplus \oplus \ominus \ominus$ | Reorder nodes **12354**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 155 | $+ \oplus \oplus \ominus +$ | **Impossible.** See #39. |
| 156 | $+ \oplus \oplus \ominus \oplus$ | Reorder nodes **12354**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 157 | $+ \oplus \oplus + \ominus$ | **Impossible.** See #32. |
| 158 | $+ \oplus \oplus + +$ | **Impossible.** See #123. |

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| 159 | $+ \oplus \oplus + \oplus$ | Reorder nodes **13245**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 160 | $+ \oplus \oplus \oplus \ominus$ | **Impossible.** See #54. |
| 161 | $+ \oplus \oplus \oplus +$ | **Impossible.** See #123. |
| 162 | $+ \oplus \oplus \oplus \oplus$ | Reorder nodes **13524**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 163 | $\oplus \ominus \ominus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 164 | $\oplus \ominus \ominus \ominus +$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 165 | $\oplus \ominus \ominus \ominus \oplus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 166 | $\oplus \ominus \ominus + \ominus$ | **Impossible.** See Section 3.2. |
| 167 | $\oplus \ominus \ominus + +$ | Reorder nodes **13245**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 168 | $\oplus \ominus \ominus + \oplus$ | Reorder nodes **13245**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 169 | $\oplus \ominus \ominus \oplus \ominus$ | **Impossible.** See Section 3.2. |
| 170 | $\oplus \ominus \ominus \oplus +$ | Reorder nodes **13425**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 171 | $\oplus \ominus \ominus \oplus \oplus$ | **Impossible.** See #27. |
| 172 | $\oplus \ominus + \ominus \ominus$ | **Impossible.** See Section 3.2. |
| 173 | $\oplus \ominus + \ominus +$ | **Impossible.** See #32. |
| 174 | $\oplus \ominus + \ominus \oplus$ | **Impossible.** See #36. |
| 175 | $\oplus \ominus + + \ominus$ | **Impossible.** See #39. |

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| 176 | $\oplus\ominus+++$ | **Impossible.** See #39. |
| 177 | $\oplus\ominus++\oplus$ | **Impossible.** See #39. |
| 178 | $\oplus\ominus+\oplus\ominus$ | **Impossible.** See #39. |
| 179 | $\oplus\ominus+\oplus+$ | Reorder nodes **13425**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 180 | $\oplus\ominus+\oplus\oplus$ | **Impossible.** See #54. |
| 181 | $\oplus\ominus\oplus\ominus\ominus$ | **Impossible.** See Section 3.2. |
| 182 | $\oplus\ominus\oplus\ominus+$ | **Impossible.** See #39. |
| 183 | $\oplus\ominus\oplus\ominus\oplus$ | **Impossible.** See #63. |
| 184 | $\oplus\ominus\oplus+\ominus$ | **Impossible.** See #39. |
| 185 | $\oplus\ominus\oplus++$ | Reorder nodes **13425**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 186 | $\oplus\ominus\oplus+\oplus$ | Reorder nodes **13245**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 187 | $\oplus\ominus\oplus\oplus\ominus$ | **Impossible.** See #63 |
| 188 | $\oplus\ominus\oplus\oplus+$ | Reorder nodes **12543**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 189 | $\oplus\ominus\oplus\oplus\oplus$ | **Impossible.** See #81. |
| 190 | $\oplus+\ominus\ominus\ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 191 | $\oplus+\ominus\ominus+$ | Reorder nodes **12435**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 192 | $\oplus+\ominus\ominus\oplus$ | Reorder nodes **12435**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 193 | $\oplus+\ominus+\ominus$ | **Impossible.** See #32. |
| 194 | $\oplus+\ominus++$ | **Impossible.** See #32. |
| 195 | $\oplus+\ominus+\oplus$ | **Impossible.** See #32. |
| 196 | $\oplus+\ominus\oplus\ominus$ | **Impossible.** See #39. |

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| 197 | $\oplus + \ominus \oplus +$ | Reorder nodes **12435**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 198 | $\oplus + \ominus \oplus \oplus$ | **Impossible.** See #54. |
| 199 | $\oplus + + \ominus \ominus$ | Reorder nodes **12354**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 200 | $\oplus + + \ominus +$ | **Impossible.** See #32. |
| 201 | $\oplus + + \ominus \oplus$ | **Impossible.** See #39. |
| 202 | $\oplus + + + \ominus$ | **Impossible.** See #39. |
| 203 | $\oplus + + + +$ | **Impossible.** See #123. |
| 204 | $\oplus + + + \oplus$ | **Impossible.** See #123. |
| 205 | $\oplus + + \oplus \ominus$ | Reorder nodes **12534**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |
| 206 | $\oplus + + \oplus +$ | Reorder nodes **12534**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 207 | $\oplus + + \oplus \oplus$ | **Impossible.** See #123. |
| 208 | $\oplus + \oplus \ominus \ominus$ | Reorder nodes **14235**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 209 | $\oplus + \oplus \ominus +$ | Reorder nodes **14235**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 210 | $\oplus + \oplus \ominus \oplus$ | Reorder nodes **12435**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 211 | $\oplus + \oplus + \ominus$ | Reorder nodes **14325**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |

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| 212 | $\oplus + \oplus + +$ | Reorder nodes **13425**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 213 | $\oplus + \oplus + \oplus$ | Reorder nodes **14325**; $\Lambda$ has two adjacent pairs of problematic determinants. All other re-embeddings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 214 | $\oplus + \oplus \oplus \ominus$ | Reorder nodes **14325**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 215 | $\oplus + \oplus \oplus +$ | Reorder nodes **12435**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 216 | $\oplus + \oplus \oplus \oplus$ | Reorder nodes **13524**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 217 | $\oplus \oplus \ominus \ominus \ominus$ | There is a recoverable **annular planar network** with this $\Lambda$. See Section 3.1. |
| 218 | $\oplus \oplus \ominus \ominus +$ | Reorder nodes **12435**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 219 | $\oplus \oplus \ominus \ominus \oplus$ | **Impossible.** See #27. |
| 220 | $\oplus \oplus \ominus + \ominus$ | **Impossible.** See #36. |
| 221 | $\oplus \oplus \ominus + +$ | **Impossible.** See #39. |
| 222 | $\oplus \oplus \ominus + \oplus$ | **Impossible.** See #54. |
| 223 | $\oplus \oplus \ominus \oplus \ominus$ | **Impossible.** See #63. |
| 224 | $\oplus \oplus \ominus \oplus +$ | Reorder nodes **12435**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 225 | $\oplus \oplus \ominus \oplus \oplus$ | **Impossible.** See #81. |

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| 226 | $\oplus\oplus+\ominus\ominus$ | Reorder nodes **12354**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 227 | $\oplus\oplus+\ominus+$ | **Impossible.** See #32. |
| 228 | $\oplus\oplus+\ominus\oplus$ | **Impossible.** See #54. |
| 229 | $\oplus\oplus++\ominus$ | **Impossible.** See #39. |
| 230 | $\oplus\oplus+++$ | **Impossible.** See #123. |
| 231 | $\oplus\oplus++\oplus$ | **Impossible.** See #123. |
| 232 | $\oplus\oplus+\oplus\ominus$ | Reorder nodes **12354**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 233 | $\oplus\oplus+\oplus+$ | Reorder nodes **12543**; $\Lambda$ has two adjacent pairs of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 234 | $\oplus\oplus+\oplus\oplus$ | Reorder nodes **13524**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 235 | $\oplus\oplus\oplus\ominus\ominus$ | **Impossible.** See #27. |
| 236 | $\oplus\oplus\oplus\ominus+$ | **Impossible.** See #54. |
| 237 | $\oplus\oplus\oplus\ominus\oplus$ | **Impossible.** See #81. |
| 238 | $\oplus\oplus\oplus+\ominus$ | **Impossible.** See #54. |
| 239 | $\oplus\oplus\oplus++$ | **Impossible.** See #123. |
| 240 | $\oplus\oplus\oplus+\oplus$ | Reorder nodes **13524**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 241 | $\oplus\oplus\oplus\oplus\ominus$ | **Impossible.** See #81. |
| 242 | $\oplus\oplus\oplus\oplus+$ | Reorder nodes **13524**; $\Lambda$ has one pair of problematic determinants. All other reorderings are clean. Therefore, the reordered network is $\Lambda$-equivalent to a recoverable **annular planar network**. |
| 243 | $\oplus\oplus\oplus\oplus\oplus$ | Reorder nodes **13524**; network becomes $\Lambda$-equivalent to a **circular planar network**. See Section 2.2. |