## Introduction

Any body in space has a gravitational force, and hence potential, associated with it. If we imagine a body, D, of mass m in $R^3$, then at any point in space outside of D there exists a gravitational field vector, $-\vec{\nabla}u$, and gravitational potential, u, due to D. D creates a vector field in 3-space according to $\int_{Source} \frac{q}{r^2}$ (q = mass of element, r = distance to element).

# ILLUSTRATION

**A vector field in $R^3$.**

What can this vector field tell us about D?

The newtonian potential, $u(\vec{x})$, obeys Poisson's equation:

$$\Delta u = -4\pi \rho(\vec{x}) \vec{\chi}(D)$$

where $\rho(\vec{x})$ is the density of D and $\vec{\chi}(D)$ is the characteristic function of the point set D. If we assume constant density, the inverse problem is to determine the function $\chi(D)$ from the known solution to the above equation. More specifically, if we measure u at a finite number of points on a surface outside of D, how can we find D? This is the inverse problem of the theory of potential.

*In this paper, I discuss the two-dimensional, discrete* version of the gravitational problem. u is measured at a finite number of points on the boundary of an (n x n) grid on which D lies.

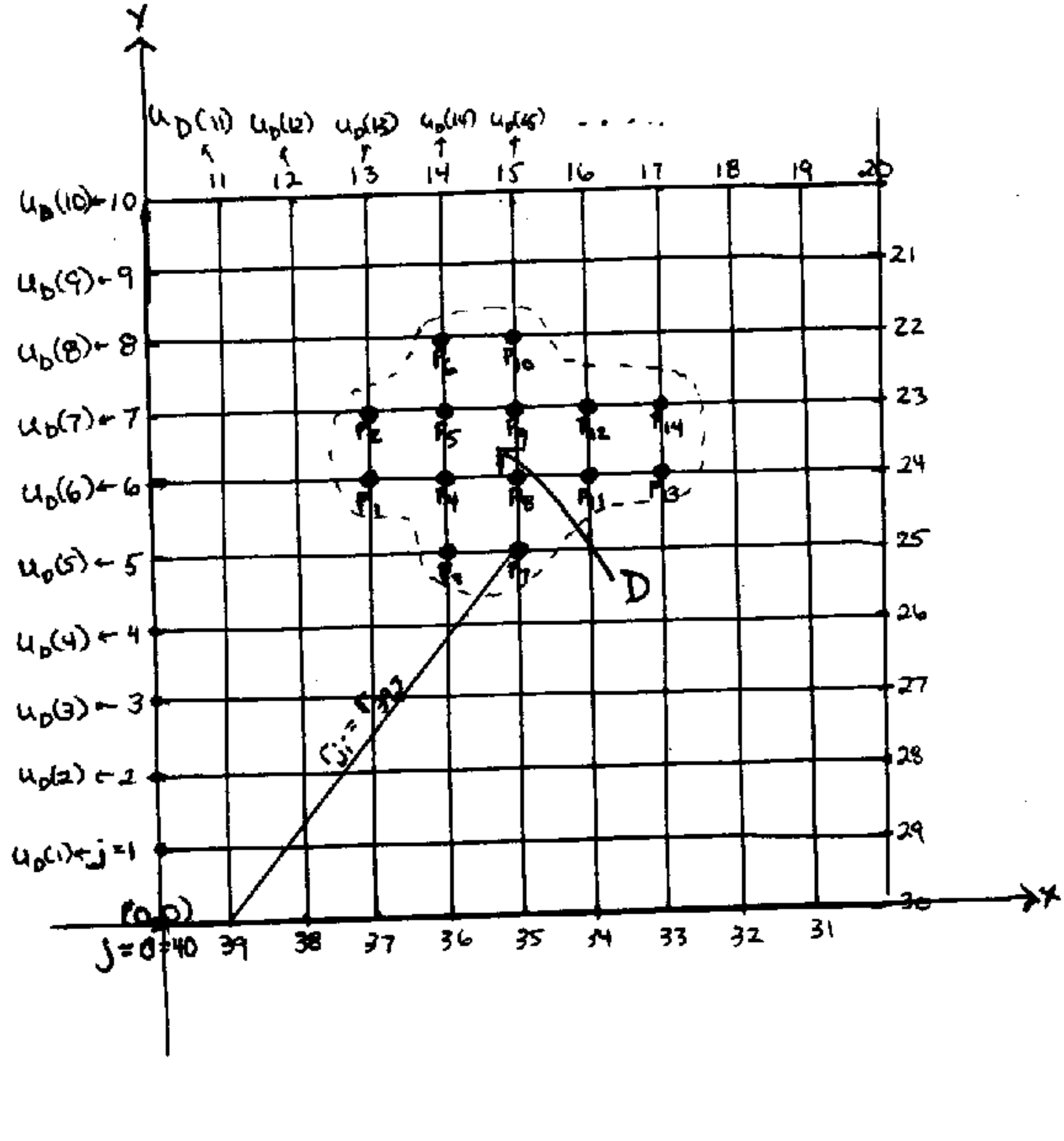
## Basic Terminology

Let $\Omega$ be an (n x n) grid on $R^2$. (That is, $\Omega$ is made up of the line segments: $x=0, x=1, x=2, \ldots, x=n$ where $y: 0 \to n$, and $y=0, y=1 \ldots y=n$, $x: 0 \to n$.) D is a set of particles (masses, fixed electromagnetic charge sources, etc.), located in a subset of the interior lattice points of $\Omega$. (Legal subsets described below.) The size of D is m, (indicating the mass), so that $D = \bigcup_{i=1}^{m} p_i$ where each $p_i$ is located at point in $R^2$, $(x_i', y_i')$. (Because $p_i$ is on a lattice point of the grid, $x_i'$ and $y_i'$ are integers such that $0 < x_i', y_i' < n$, for each i.) The $p_i$ are labelled from bottom to top, then left to right. u is measured at the set of points labelled {1,2,3,...,4n} which lie on the boundary of $\Omega$. A point in this set will be referred to as j, and $j = (x_j, y_j)$. (This is to be consistent with textbook theory notation which measures potential from a particle located at $(x', y')$ at a point in space $(x, y)$.) $x_j$ and $y_j$ are integers such that $0 < x_j, y_j < n$, for each j. Let $j_0 = (0, 0) = j_{4n}$. These points are then labelled clockwise around $\Omega$ starting at j .

# LAYTECH GRAPHIC I

**Set up for n= 10 case.**

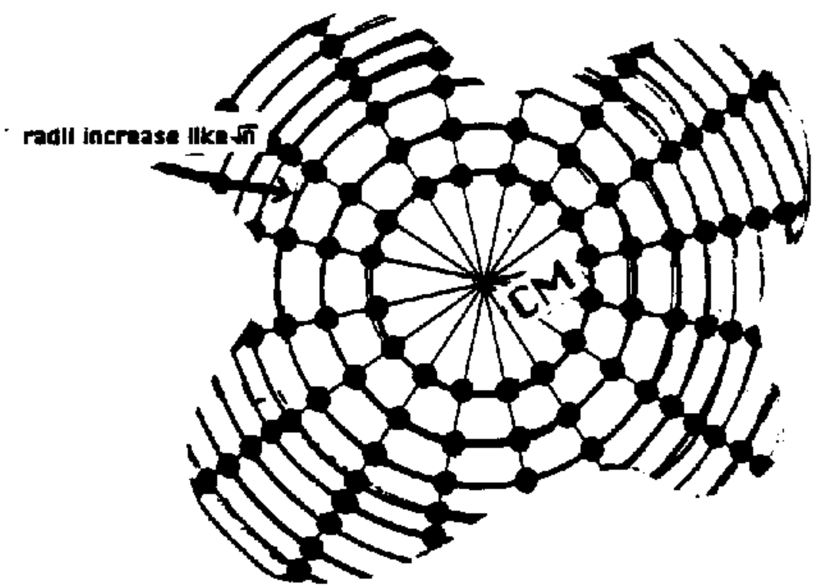
Sketch of Lautech Graphic 1



$u_i(j)$ is the potential measured at j due to $p_i$. $r_{j,i}$ is the distance from j to $p_i$; that is, $r_{j,i} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$. $u_D(j)$ represents the potential at j due to the whole distribution D.

## Assumptions

Assume D has constant density . In some potential theory, determining the density of D is part of the inverse problem, but I don't attempt that here. Constant density can be represented on Ω by assigning each $p_i$ the value 1. If D were star-shaped and we got measurements on a ball enclosing D, constant density could be represented in another way:




Additionally, assume that for any subset of points of D which lie on the same line x= c (i.e. are in the same collumn), the y coordinates of those points form a contiguous set of integers, and for a subset on the same line y= c (in the same row), the x coordinates of those points form a contiguous set of integers. This means D has no holes. Examples of legal configurations of D are as follows:

# LAYTECH GRAPHIC II

(6) and (7) are not legal.

This criterion is the rectangular analogy to "star-shaped", and is assumed to be necessary due to Gauss' law. For if $D_1$ = a spherical shell of mass m centered at the same point as $D_2$ = a solid sphere of mass m, then $u_{D_1}(\vec{x}) = u_{D_2}(\vec{x})$ although $\chi(D_1) = \chi(D_2)$. (The discrete representations of $D_1$ and $\vec{D}_2$ on $\Omega$ would give similar u(j)'s, and $\lim_{n \to \infty} u_{D_1}(j) = u_{D_2}(j)$.) Other non-star-shaped D's may also give the same $\vec{u}$, but it is in general true only for D with spherical symettry.

Also, consider that the zero potential occurs consistently, infinately far from D.

## The Forward Problem

The field due to a distribution of particles can be reguarded as the superposition of the fields from the individual particles. This principle works for u as well as $-\vec{\nabla}u$, therefore we add the individual potentials, i.e. in general

$$u(\vec{x}) = \int_{\text{all sources}} \frac{\rho(x',y',z')\,dx'dy'dz'}{r}$$

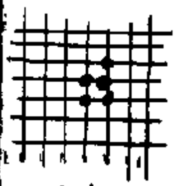
where r is the distance from element dx'dy'dz' to $\vec{x}$ at which u is evaluated, and $\rho$ is the density function. With this in mind, we know for D as defined on $\Omega$:

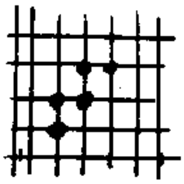
$$u_D(j) = \sum_{i=1}^{m} \log\left[\frac{1}{r_{ji}}\right] \qquad \text{for each j.}$$

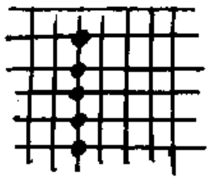
(Logarithms compensate for 2 dimensions).

Given any n and D, then, u can be calculated as follows:

Sketch of Laytech Graphic II


(1)


(2)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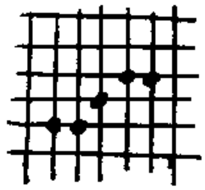
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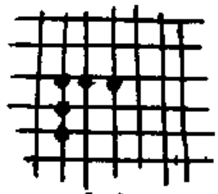
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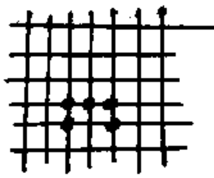
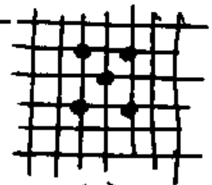

(6)


(7)

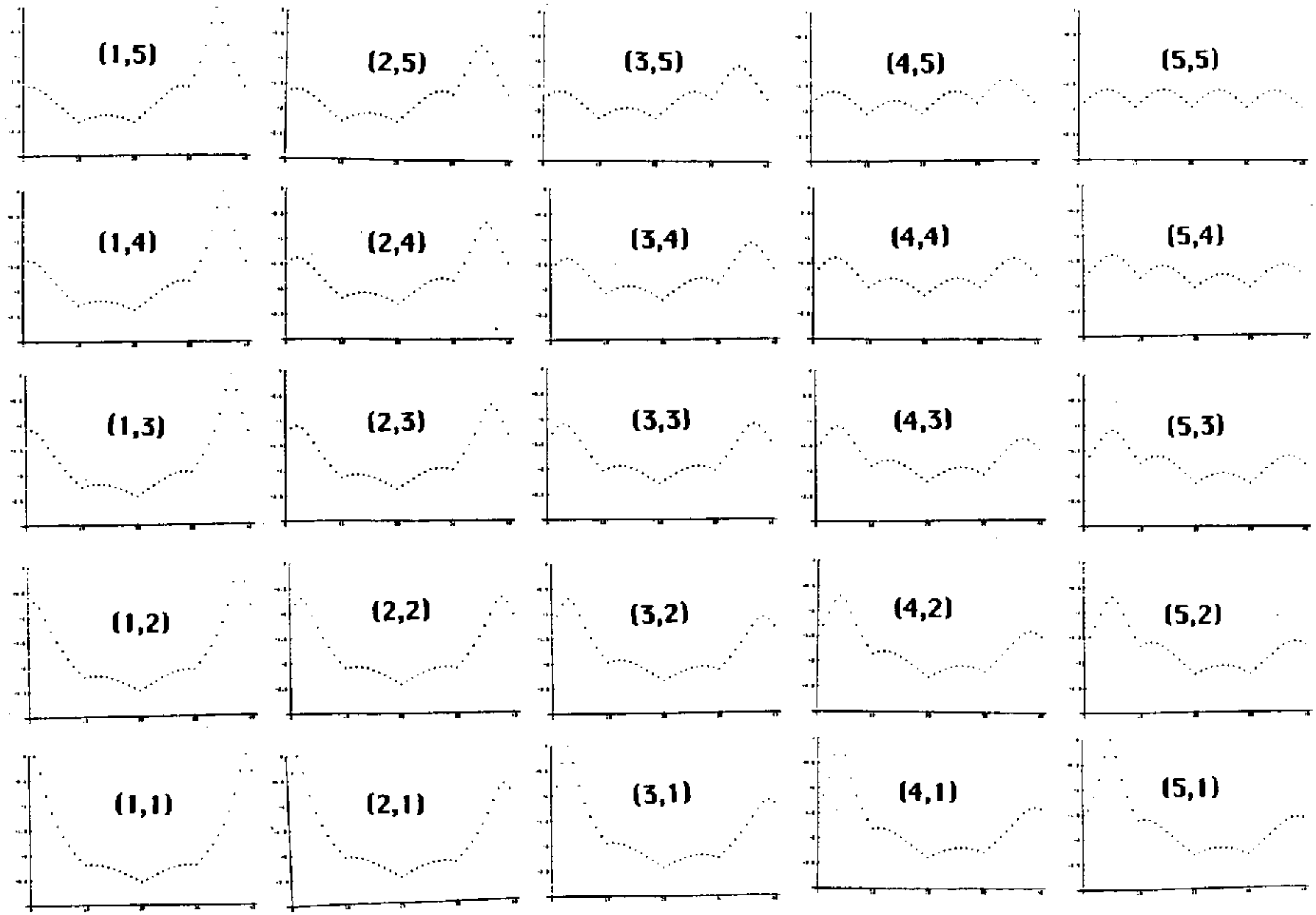
$$u_D(j) = \sum_{i=1}^{m} \log\left[\frac{1}{\sqrt{x_i'^2 + (y_i' - j)^2}}\right] \quad \text{for} \quad 0 < j \leq n$$

$$= \sum_{i=1}^{m} \log\left[\frac{1}{\sqrt{(x_i' + n - j)^2 + (n - y_i')^2}}\right] \quad \text{for} \quad n < j \leq 2n$$

$$= \sum_{i=1}^{m} \log\left[\frac{1}{\sqrt{(n - x_i')^2 + (y_i' - 3n + j)^2}}\right] \quad \text{for} \quad 2n < j \leq 3n$$

$$= \sum_{i=1}^{m} \log\left[\frac{1}{\sqrt{(x_i' - 4n + j)^2 + y_i'^2}}\right] \quad \text{for} \quad 3n < j \leq 4n$$

I wrote computer codes which give a numerical table of values for u at each j, and graph j vs. u, once assigned a D.




j vs. u (j) for D of m=1 located at pt.s on (10x10) grid as labelled