Recovering Conductors in Circular Networks with Interior Source Currents

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Abstract

A detailed algorithm is given for recovering circular networks with $m = 1$, $n = 4$. The method is generalized for recovering circular networks with $m = 1$, $n \geq 5$. A less rigorous discussion gives modifications which allow for the recovery of networks with $m = 2$, $n \geq 6$. A conjecture is made conerning circular networks with $m \geq 3$.

1. Introduction

We will examine circular networks of the form $C_1(m, n)$, made up of m circles, n rays, and a strictly positive real-valued function γ, called the conductivity, which takes on one and only one value for each resistor in the network (see Curtis, Mooers and Morrow [1]). In particular we will look at these networks under the following circumstances. Potentials on the boundary are fixed at zero, and the n boundary resistors have unit conductance. Some (perhaps negative) amount of current flows into each interior node, and flows out of each boundary node. In studying such networks, it is important to recall Ohm's Law (which defines current) and Kirchhoff's Law (which states that the total current flowing into a node, excluding source current, is zero).

Ohm's Law. Let I denote the current flowing from node q to node p in a circular network of form $C_1(m, n)$. Let $u(p)$ denote the potential at p and let $u(q)$ denote the potential at q. Let $\gamma(p, q)$ denote the conductance of the resistor connecting nodes p and q. Then

$$I = \gamma(p, q)(u(q) - u(p)).$$

KIRCHHOFF'S LAW. Suppose there is source current I_p flowing into a node p in a circular network of form $C_1(m, n)$. Let $N(p)$ denote the set of nodes q neighboring p (connected to p by a single resistor) and let $\gamma(p, q)$, $u(p)$, and $u(q)$ denote the same quantities as in the statement of Ohm's Law. Then

$$\sum_{q \in N(p)} \gamma(p, q)(u(p) - u(q)) = I_p.$$

Using Kirchhoff's Law at each interior node, and accounting for the interior source currents, we create a system of $mn+1$ linear equations in $mn+1$ unknowns, which can be written as the matrix equation $Ku = b$ (again, see [1] for a more thorough exposition), where u is the vector of interior potentials. After solving for u, the current flowing out at each boundary node can be determined quite easily using Ohm's Law. In fact, the current[1] at a boundary node will simply be the negative of the potential at that node's interior neighbor (since boundary potentials are 0 and boundary conductivities are 1).

With i ranging from 1 to $mn+1$, we set up experiments ($mn+1$ of them) in which unit source current flows into the ith interior node and zero source current flows into the remaining mn interior nodes. Note that any source current configuration is a linear combination of these $mn+1$ configurations; so in a sense, they are basis vectors for the space of source current arrangements, and we will use them in precisely this way. We can solve for the currents flowing out at the boundary in each of the $mn + 1$ cases, and construct an $n \times (mn+1)$ matrix A such that $A_{i,j}$ is the current flow at the ith boundary node when there is a unit source current at the jth interior node and zero source current at the other interior nodes. This matrix A maps interior source currents to boundary currents in the following way: multiplying A on the right by a vector of interior source currents yields a vector of boundary currents. The inverse problem is, given the matrix A and the fixed boundary information mentioned above, to recover the conductivity function γ of the network.

We will discuss in detail the solution of the inverse problem in the case $m = 1$, $n = 4$, and outline a generalization of the algorithm which can be

[1]The convention in this paper will be to take inflowing current as positive and outflowing current as negative. When we speak simply of the current "at" a particular node, we are referring to the inflowing current.

applied to any network in which $m = 1$ and $n \geq 4$. We will also examine networks with $m = 2$ and describe a method for recovering the conductivity of such networks for $n \geq 6$. Many of these recoveries are not possible using the Dirichlet-to-Neumann map Λ (detailed in [1]), which takes boundary potentials to boundary currents in a network without interior source currents.

2. Recovery of a Network with One Circle and Four Rays

We will examine networks of the form $C_1(1, 4)$ (see Figure 1 below), with one circle and four rays. In Figure 1, parenthetical labels represent conductances while the numbers lacking parentheses index the interior nodes. The four boundary resistors have conductance 1, as shown, and the remaining eight resistors have conductances γ_i $(1 \leq i \leq 8)$ — the quantities we wish to recover. In general $u(p)$ will denote the potential at node p, where the interior nodes are numbered as in the diagram. Boundary nodes have potential fixed at zero. There are five interior nodes in this network, and thus our current-to-current matrix A will be a 4×5 matrix.

Characterization of K^{-1}

Note that the presence of interior source currents does not alter the Kirchhoff matrix K in the equation $Ku = b$; only the entries of b are changed. In fact, the ith entry of b is the current flowing in at the ith interior node. The five source current configurations used for constructing A (as discussed previously) correspond to $b = e_j$, $1 \leq j \leq 5$, where e_j is the jth column of the 5×5 identity matrix. Solving $Ku_j = e_j$ yields $u_j = K^{-1}e_j$, so $K^{-1} = \begin{pmatrix} u_1 & u_2 & u_3 & u_4 & u_5 \end{pmatrix}$. This means that each column of K^{-1} is a column of interior potentials resulting from a certain configuration of interior source currents. In particular, $K^{-1}_{i,j}$ is the potential at the ith interior node due to a unit source current at the jth interior node and zero source current elsewhere.

Recall that the current flow into a boundary node is simply the negative of the potential at its interior neighbor, which means that $K^{-1}_{i,j} = -A_{i,j}$ for $1 \leq i \leq 4$ and $1 \leq j \leq 5$. Furthermore, K^{-1} is symmetric (because K is symmetric). So we can characterize K^{-1}, in terms of A, as follows:

$$K^{-1} = \begin{pmatrix} -A_{1,1} & -A_{1,2} & -A_{1,3} & -A_{1,4} & -A_{1,5} \\ -A_{2,1} & -A_{2,2} & -A_{2,3} & -A_{2,4} & -A_{2,5} \\ -A_{3,1} & -A_{3,2} & -A_{3,3} & -A_{3,4} & -A_{3,5} \\ -A_{4,1} & -A_{4,2} & -A_{4,3} & -A_{4,4} & -A_{4,5} \\ -A_{1,5} & -A_{2,5} & -A_{3,5} & -A_{4,5} & ? \end{pmatrix},$$

where all but one entry of K^{-1} is known.

Multiplying K^{-1} on the right by a vector of interior source currents yields a vector of interior potentials arising from those source currents. Because $K^{-1}_{5,5}$ is unknown, we cannot compute the potential at node 5 when there is source current flowing in there; however, multiplying K^{-1} on the right by a vector of currents

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ 0 \end{pmatrix},$$

where i_1, i_2, i_3, and i_4 are any numbers, will yield a vector of potentials arising from putting in a current of i_1 at node 1, i_2 at node 2, i_3 at node 3, i_4 at node 4, and zero current into node 5. We capitalize on this idea for our next result.

Setting Up Special Potential Arrangements

Lemma 1. *Consider a circular network of form $C_1(1,4)$, with nodes numbered as in Figure 1. There exist unique numbers x_1, x_2, and x_3 such that nodes 1, 4, and 5 each have unit potential when a source current of x_i flows in at node i $(1 \le i \le 3)$ and zero source current flows in at nodes 4 and 5. Furthermore, there exist unique numbers y_2, y_3, and y_4 such that nodes 1, 2, and 5 each have unit potential when a source current of y_i flows in at node i $(2 \le i \le 4)$ and zero source current flows in at nodes 1 and 5.*

Proof. We wish to show that the two potential arrangements shown below (Configurations A and B) can be established by putting source currents at the designated nodes. In the diagrams, source currents are written inside square brackets and potentials appear without them. Open circles drawn on resistors indicate that no current is flowing there.

As stated earlier, multiplying K^{-1} on the right by a vector of interior source currents yields a vector of interior potentials due to those source currents. So in seeking a way to set up Configuration A and Configuration B, we are looking for unique solutions to the following matrix equations:

$$K^{-1}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ u(2) \\ u(3) \\ 1 \\ 1 \end{pmatrix}, \qquad K^{-1}\begin{pmatrix} 0 \\ y_2 \\ y_3 \\ y_4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ u(3) \\ u(4) \\ 1 \end{pmatrix},$$

where K^{-1} is the matrix shown on page 4 (with one entry unknown). Because we are not interested in the two non-unit potentials of each arrangement ($u(2)$ and $u(3)$ in Configuration A, $u(3)$ and $u(4)$ in Configuration B), we can eliminate those rows from the righthand side and from K^{-1}. Moreover, since two of our source currents are zero in each case (nodes 4 and 5 in Configuration A, nodes 1 and 5 in configuration B), we can also eliminate the zero entries from our two source current vectors, and the appropriate columns (4 and 5 in Configuration A, 1 and 5 in Configuration B) from K^{-1}. We now have:

$$\begin{pmatrix} -A_{1,1} & -A_{1,2} & -A_{1,3} \\ -A_{4,1} & -A_{4,2} & -A_{4,3} \\ -A_{1,5} & -A_{2,5} & -A_{3,5} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} -A_{1,2} & -A_{1,3} & -A_{1,4} \\ -A_{2,2} & -A_{2,3} & -A_{2,4} \\ -A_{2,5} & -A_{3,5} & -A_{4,5} \end{pmatrix}\begin{pmatrix} y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{2}$$

If we can show that the two 3×3 submatrices of K^{-1} shown above are nonsingular, then the proof will be complete.

Consider the first equation above (corresponding to Configuration A), but with a column of 0's, rather than 1's, on the righthand side. We know there is no source current at node 4, and that potentials are 0 at node 4 and at three of its four neighbors (all but node 3); therefore by Kirchhoff's Law, node 3 must also have potential 0. Now we know that three neighbors of node 5 (nodes 1, 3, 4), as well as node 5 itself, have potential 0. Since there is no source current at node 5, Kirchhoff's Law dictates that the potential must be 0 at node 2 as well. We have now established that the potential is 0 at every node in the network, and thus no current can flow along any resistor. Therefore the three source currents must be 0. By rotation, an analogous argument can be applied to the second equation above (corresponding to Configuration B). We have shown that these homogeneous systems have only the trivial solution, which implies that the submatrices of K^{-1} are indeed nonsingular, and the above equations have unique solutions. **Q.E.D.**

Recovering the Circular Conductances

We are now prepared to recover the circular conductances of the network, utilizing Lemma 1. The following theorem gives explicit formulas for these conductances.

THEOREM 1. *Consider a circular network of form $C_1(1,4)$, with nodes and conductances labelled as in Figure 1, and let $x = (\ x_1 \quad x_2 \quad x_3\ )$ and $y = (\ y_2 \quad y_3 \quad y_4\ )$ be the solutions (1) and (2), respectively. Then γ_1, γ_2, γ_3, and γ_4 are uniquely determined, and*

$$\gamma_1 = \frac{x_1 - 1}{1 + x_1A_{2,1} + x_2A_{2,2} + x_3A_{2,3}},$$

$$\gamma_2 = \frac{y_2 - 1}{1 + y_2A_{3,2} + y_3A_{3,3} + y_4A_{3,4}},$$

$$\gamma_3 = \frac{-1}{1 + x_1A_{3,1} + x_2A_{3,2} + x_3A_{3,3}},$$

$$\gamma_4 = \frac{-1}{1 + y_2A_{4,2} + y_3A_{4,3} + y_4A_{4,4}}.$$

Proof. Consider Configuration A. We know that a current of 1 flows out at the boundary node neighboring node 1. Since no current flows between nodes 1 and 3 or between nodes 1 and 5, the current flowing from node 1 to node 2 must be $x_1 - 1$. Ohm's Law gives the equation

$$x_1 - 1 = \gamma_1(1 - u(2)), \tag{3}$$

where $u(2)$ is the potential at node 2. This potential is the negative of the current flow at the boundary node neighboring node 2, which is the second row of

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \\ 0 \end{pmatrix}.$$

So $u(2) = -(x_1 A_{2,1} + x_2 A_{2,2} + x_3 A_{2,3})$, and we can solve for γ_1. Doing so yields the formula above.

Note that, in order to solve for γ_1, we must divide by the quantity $1 - u(2)$. It would be worthwhile, then, to show that this quantity cannot be zero. If the potential at node 2 were 1, then applying Kirchhoff's Law to the center node (node 5) would indicate that the potential at node 3 was also 1. But if node 3 had potential 1, then Kirchhoff's Law would be violated at node 4, which would have three neighbors with potential 1 and one neighbor with potential 0. Since node 4 has potential 1 itself, and has no source current, this situation would not be possible. Therefore by contradiction, node 2 cannot have potential 1, and we need not worry about dividing by zero to recover γ_1.

Solving for γ_3 is done the same way, except for the absence of a source current at node 4, which makes our equation

$$-1 = \gamma_3(1 - u(3)). \tag{4}$$

We can easily evaluate $u(3)$ just as we did $u(2)$, and we have already shown that it cannot equal 1, so we solve for γ_3, obtaining the formula given in the statement of Theorem 1.

Using Configuration B, we recover γ_2 and γ_4 in a manner analogous to that used in the recovery of γ_1 and γ_3. Each conductance is expressed by an explicit formula, which shows uniqueness of the solution. **Q.E.D.**

Recovering the Radial Conductances

Once the circular conductances are known, it is fairly easy to recover the radial conductances. The following theorem gives an explicit formula for them.

THEOREM 2. *Consider a circular network of form $C_1(1,4)$, with nodes numbered as in Figure 1, and let addition be modulo 4. For $1 \leq i \leq 4$ and $1 \leq j \leq 4$, let $u(i)$ denote the potential at node i, let $\gamma(i,j)$ denote the conductance of the resistor connecting nodes i and j, and let x_i denote the source current put into the network at node i. Then for node p, $1 \leq p \leq 4$, $\gamma(p,5)$ is uniquely determined and*

$$\gamma(p,5) = \frac{x_p - u(p) - \gamma(p,p+1)(u(p)-u(p+1)) - \gamma(p,p-1)(u(p)-u(p-1))}{u(p)-u(5)}$$

if and only if $u(p) \neq u(5)$. Moreover, for each p, $1 \leq p \leq 4$, there exists a set of source currents x_i, $1 \leq i \leq 4$ (no source current at node 5), such that the condition is satisfied and the above formula is correct.

Proof. The necessity of the condition $u(p) \neq u(5)$ is obvious, since otherwise the formula would be undefined and could not possibly represent a conductance or any quantity for that matter.

The sufficiency of the condition is fairly easy to show, as well. We write Kirchhoff's Law at node p. Only one quantity in the equation, namely $\gamma(p,5)$ is unknown. Moving everything else to the righthand side leaves us with the formula above.

Now we must show that a vector x of source currents for nodes 1–4 can be selected such that $u(p) \neq u(5)$, or, in other words, such that there is a nonzero current flowing between nodes p and 5. Our formula would still be correct if source current were put in at node 5, but then we would not know the value of $u(5)$ (remember we are missing the last entry of K^{-1}), and thus would not be able to solve explicitly for $\gamma(p,5)$.

We seek a solution to the matrix equation

$$K^{-1}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ 0 \end{pmatrix} = \begin{pmatrix} u(1) \\ u(2) \\ u(3) \\ u(4) \\ u(5) \end{pmatrix}, \tag{5}$$

where K^{-1} is the matrix shown on page 4, and where, on the righthand side, $u(p) \neq u(5)$, whichever number p happens to be.

Suppose we remove the last digit of the source current vector (the only known digit, 0) and remove the last column from K^{-1}. Next, suppose we remove row r from K^{-1} and the rth entry from the potential vector on the righthand side, where $1 \leq r \leq 4$ and $r \neq p$. Solving this new equation is equivalent, for our purposes, to solving (5), but now we can show that a solution exists for any righthand side by showing the 4×4 submatrix of K^{-1} to be nonsingular.

We can show nonsingularity by showing that the homogeneous system has only the trivial solution. Suppose all the potentials on the righthand side are 0. Now all the nodes in the network, except one (a non-center, non-boundary node), are known to have potential 0. That last node must also have potential 0, because if it didn't, current would flow between it and node 5, which would violate Kirchhoff's Law at node 5 (since there is no source current there). With all potentials 0, no current can flow anywhere in the network, so all the source currents must also be 0.

The submatrix of K^{-1} is nonsingular, therefore we can set up an arrangement of source currents such that, for each p ($1 \leq p \leq 4$), $u(p) \neq u(5)$. Therefore we can use the formula given in the statement of Theorem 2 to solve for each of the radial conductances. The presence of an explicit formula guarantees uniqueness, and the proof is complete. **Q.E.D.**

3. Networks with One Circle and Five or More Rays

The method for solving networks of the form $C_1(1, n)$, for $n \geq 5$, is analogous to the one we just detailed for solving those of the form $C_1(1, 4)$. The idea is to recover each circular conductance by setting three potentials equal, thereby eliminating certain current flows, just as we did in Configuration A and Configuration B. A network with more than four rays will require more than two such configurations in order to find every circular conductance, but by the symmetry of the network, proving that one such arrangement exists is equivalent to proving that they all do.

Here is how, in general, we go about proving that we can set up the desired potential configurations. We begin with the equation

$$K^{-1}x = u, \tag{6}$$

where x is the vector of source currents and u is the vector of interior potentials. We decide that only three entries of x will be nonzero (and those three must not include the last entry, representing the center node), and then, like before, we eliminate rows and columns of K^{-1}, and the corresponding entries of x and u, until we are left with a 3×3 submatrix of K^{-1}, a source current vector of three elements, and a column of three 1's on the righthand side.

Our only task now is to show that this submatrix of K^{-1} is nonsingular, and we do so once again by proving that the homogeneous system has only the trivial solution. The network has potential 0 at the center node, at two adjacent nodes on the circle, and at every boundary node. Using Kirchhoff's Law (assuming no source current) at one of those two circle-residing nodes, we show the potential to be 0 at the next node on the circle. We move to that node and apply Kirchhoff's Law again, then continue on around the circle (either clockwise or counterclockwise, depending on which node we begin with) until only one node remains whose potential is not known to be 0. Applying Kirchhoff's Law at the center node makes the potential 0 at this last node as well, and with all potentials 0, no current can flow, and all source currents must be 0 as well. This method always leaves three nodes on the circle at which Kirchhoff's Law was not used, and it is at these three nodes that the source currents are put in. If the three nodes which share the same potential are numbered i, $i+1$, and $n+1$ ($1 \leq i \leq n$ and node $n+1$ is the center node), then the three nodes which possess source current will be nodes i, $i-1$, and $i-2$, where addition (and subtraction) is modulo n.

The radial conductances are also recovered in a manner very similar to the way they were recovered in the case of four rays. The formula given in Theorem 2 is identical for higher values of n, except that the quantity $u(5)$ must be replaced with the quantity $u(n+1)$. Showing that a set of source currents exists such that $u(p) \neq u(n+1)$ is accomplished in exactly the same fashion as before, except that the matrix equation is larger, and we reduce K^{-1} to an $n \times n$ submatrix, rather than a 4×4.

4. Networks with Two Circles

The method used to recover the conductivity of networks of the form $C_1(1, n)$ can be modified to work for networks of the form $C_1(2, n)$, with the condition $n \geq 6$ (it is probably impossible to recover a network with two circles and less than six rays using the information we allow ourselves here; even if it is possible, however, no such recovery is presented in this paper). We number the nodes in these networks moving clockwise and inwards. So the outer circle contains nodes 1 thru n, the inner circle contains nodes $n+1$ thru $2n$, and the center node is numbered $2n+1$.

Once again, we recover all the circular resistors and then move inwards to the adjoining set of rays. This time, however, we must recover two sets of each. Surprisingly, the technique used for the inner level is almost identical to that used for the outer level.

The first step is to recover the outer circular conductances. As with single-circle networks, we find an arrangement of source currents which will give certain nodes the same potential and thus eliminate current flow between them. This time we select five nodes, rather than three, in the following way. We take two neighboring nodes (connected by a single resistor) on the outer circle. Then we take two neighboring nodes on the inner circle, exactly one of which is connected by a radial resistor to one of the nodes we selected on the outer circle. Finally we take the center node.

We break down (6) again, eliminating from K^{-1} columns where there is no source current and rows where the potential is unimportant. We also eliminate the appropiate entries of x and u, and we end up this time with a 5×5 submatrix of K^{-1}, a column of five source currents, and a column of five 1's. Our task, then, is to show that this system has a solution by showing the indicated submatrix to be nonsingular.

For the following statements let addition be modulo n. If the nodes with unit potential are numbers $i-1$, i, $i+n$, $i+n+1$, and $2n+1$, then the five nodes with source current (there must be exactly five, to keep the submatrix of K^{-1} square so we can invert it) must be in an unbroken chain on the outer circle (for some j, they will be nodes $j, j+1, \ldots, j+4$) and must contain at least one of the two outer circle nodes with unit potential. Within these two restrictions, we may choose any five nodes to receive source current, and still show that our 5×5 submatrix of K^{-1} is nonsingular by the old homogeneous system argument. This works by setting the selected equipotential nodes to have potential 0, and then working around the network, clockwise and/or counterclockwise, using Kirchhoff's Law at nodes on both the outer and inner circles, and finally at the center node, until every node in the network has been shown to have potential 0. If the five source-current-endowed nodes are chosen strategically as outlined above, this process can be accomplished without using Kirchhoff's Law at any of them.

After proving that the desired configuration is possible, we put in the source currents which will produce it, and recover the circular conductances on each side of the outer circular resistor with no current flowing through it (using the numbering scheme of the previous paragraph, this would be the resistor between nodes $i-2$ and $i-1$ and the resistor between nodes i and $i+1$). The symmetry of the circular network allows us to set up analogous potential configurations around the network and eventually solve for all the outer circular conductances.

The outer radial conductances are computed precisely as the radial conductances were computed in networks with one circle. The formula given in Theorem 2 still holds, except that we must change the quantity $u(5)$ to the quantity $u(p+n)$. The method for showing $u(p) \neq u(p+n)$ is almost the same as the method for showing $u(p) \neq u(5)$ in the $C_1(1,4)$ network. In this case we create an $n \times n$ submatrix of K^{-1}, removing the last $n+1$ columns and a randomly chosen set of $n+1$ rows (not including row p or row $2n+1$). We also remove these rows from the potential vector, and we remove the $n+1$ 0's from the source current vector. The principle is the same, modified only to accomodate the larger geometry.

The inner circular conductances are computed just as the outer circular conductances were. We use the same nodes for current flow restriction, the same nodes for source current, and the same argument that such an arrangement is possible. The only difference is that this time we solve for the inner circular conductances rather than the outer ones. Using the same numbering system we used with the outer circle, each configuration will allow us to recover the resistor between nodes $i+n-1$ and $i+n$ and the resistor between nodes $i+n+1$ and $i+n+2$. We need to know the conductances of the outer radial resistors in order to tackle the inner circle (just as we had to know the conductances of the boundary resistors to recover the outer circle), which is why we had to compute them first.

Finally we recover the inner radial conductances, ultimately using the same method we have used on all radial resistors. The formula of Theorem

2 is still valid, with the following modifications: there is no source current at node p, so the term x_p becomes 0 and drops out; the quantity $u(p)$, where it appears alone in the numerator, becomes $\gamma(p, p-n)(u(p) - u(p-n))$ (it loses its simplified form because node p no longer borders a resistor with unit conductance and 0 potential at the neighboring node); the quantity $u(5)$ becomes $u(2n+1)$. The process of showing $u(p) \neq u(2n+1)$ is identical to the process by which we show $u(p) \neq u(p+n)$ for the outer radial resistors.

5. Networks with Three or More Circles

We conclude with a conjecture concerning the recovery of circular networks of the form $C_1(m, n)$, with $m \geq 3$.

CONJECTURE 1. *Fixing boundary potentials at* 0 *and boundary conductances at* 1, *and using the A matrix as described in this paper, the conductivity of a circular network of the form $C_1(m, n)$ can be determined uniquely, provided the condition $n \geq 2m + 2$ is true.*

As of the writing of this paper, no detailed research has been done in relation to this conjecture, but it may act as a starting point for further investigation of circular resistor networks with interior source currents.

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Reference

[1] Edward Curtis, Edith Mooers and James Morrow. "Finding the Conductors in a Circular Network from Boundary Measurements."