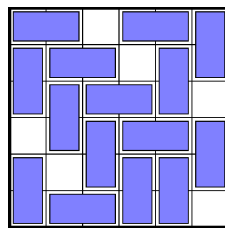


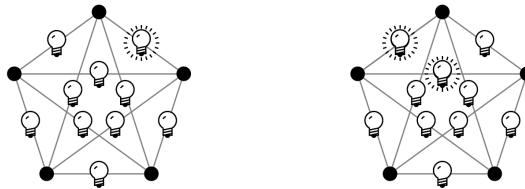
Only the Most Galvanizing Problems 1

On Wednesday April 12th, there will be a quiz in class containing one of these questions.

1. Explain why the proof that it is not possible to tile a 6×6 board without creating a fault line fails for an 8×8 board.
2. Can you tile a 4×4 board with 6 dominoes so that the four empty squares are all in different rows and columns? What about a 6×6 board with 15 dominoes? Or a 8×8 board with 28 dominoes? Or a $2m \times 2m$ board with $2m^2 - m$ dominoes? If it's possible, give an example. If it's not possible, explain why not. (For example, here is an attempt that *almost* works for the 6×6 board except for the fact that there are two empty squares in the last column.)



3. Five pillars are arranged in a circle, and between every pair of pillars is a string with a light bulb on it. You are allowed to make the following move as many times as you want: pick three pillars, then turn the three light bulbs on the strings connecting those pillars on (if they're off) or off (if they're on). For example, if you pick the top three pillars in the picture on the left, you would end up with the picture on the right. In the beginning, one light bulb is on and all the rest are off. Is it possible to turn all the light bulbs off? If so, show how. If not, explain why.



Is there a way of starting with eight light bulbs on and two off so that it is possible to turn all of them off?

4. You are driving in an infinite city whose blocks form a square lattice. At every intersection, you must turn left or right—you can't go straight. You start from the red point driving north. Is it possible to return to the red point while driving south?

