Homework 8 - Math 409

In preparation of Quiz 8 on June 1

1. Let P and Q be as defined in class for matroids. We showed that each vertex of Q is in P. Show that this implies that $Q \subseteq P$ by showing that any convex combination of vertices of Q is also in P.

2. Let $V(S) := \{C \in \mathcal{C} : C \nsubseteq S \text{ and } S \nsubseteq C\}$ be the set of chain violations as defined in class during the last proof of $P = Q$ for the matroid polytope. Carefuly show that if $T \in \mathcal{S}^* \backslash \mathcal{C}$ and $D \in V(T)$, then $|V(T \cup D)| < |V(T)|$ and $|V(T \cap D)| < V(T)$

3. At some point during the baseball season, each of n teams of the American League has already played several games. Suppose team i has won w_i games so far, and $g_{ij} = g_{ji}$ is the number of games that teams i and j have yet to play against each other. No game ends in a tie, so each game gives one point to either team and zero to the other. You would like to decide if your favorite team, say team n, can still win. In other words, you would like to determine whether there exists an outcome to the games to be played (remember, there are no ties) such that team n has at least as many victories as all the other teams (we allow team n to be tied for first place with other teams). Show that this problem can be solved as a maximum flow problem. Give a necessary and sufficient condition on the g_{ij}'s so that team n can still win.

4. Consider the following orientation problem. We are given an undirected graph $G = (V, E)$ and integer values $p(v)$ for every vertex $v \in V$. We would like to know if we can orient the edges of G such that the directed graph we obtain has at most $p(v)$ arcs incoming to v (the 'in-degree requirements'). In other words, for each edge $\{u, v\}$, we have to decide whether to orient it as (u, v) or as (v, u), and we would like at most $p(v)$ arcs to be oriented towards v. Show that the problem can be formulated as a maximum flow problem. That is, show how to create a maximum flow problem such that, from its solution, you can decide whether or not the graph can be oriented and if so, how it also gives the orientation.