Homework 7 - Math 409

In preparation of Quiz 7 on May 23

1. Let P and Q be as defined in class for matroids. Show that $P \subseteq Q$ by showing that

   1. each incidence vector of an independent set is in Q, and
   2. that any convex combination of such vectors is also in Q.

2. Let $M = (E, \mathcal{F})$ be a matroid and let $E' = \{e \in E : \{e\} \in \mathcal{F}\}$. Show that $\dim(P) = |E'|$ (where E' is the set of incidence vectors of independent sets and $P = \text{conv}(X)$ like in class) and show that the description for Q (as defined in class) has the required number of linear independent equations.

3. Let $M = (E, \mathcal{F})$ be a matroid and let P be the corresponding matroid polytope. Show that two independent sets S_1 and S_2 are adjacent on P if and only if

   1. either $S_1 \subseteq S_2$ and $|S_1| + 1 = |S_2|$,
   2. or $S_2 \subseteq S_1$ and $|S_2| + 1 = |S_1|$,
   3. or $|S_2 \backslash S_1| = |S_1 \backslash S_2| = 1$ and $S_1 \cup S_2 \notin \mathcal{F}$.

   Recall that two vertices are adjacent on P if and only if there exists an objective function c such that these two vertices are the only ones minimizing $c^\top x$ over P.