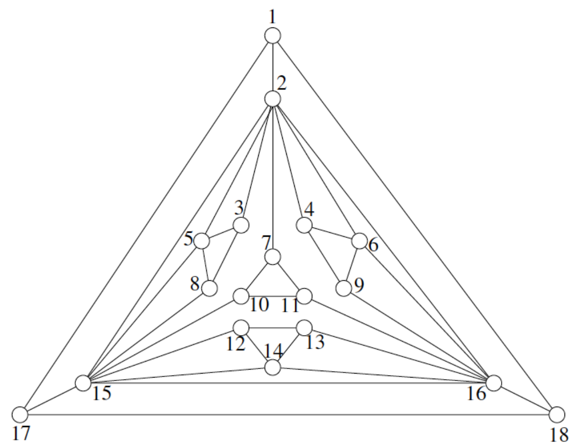
Homework 3 - Math 409

In preparation of Quiz 3 on April 18

1. Use the algorithm seen in class to find a maximum size matching (and check that it is maximum) in the graph below.



2. Give an example of a graph G, a matching M, and a blossom B for M such that a maximum matching in G/B does not lead to a maximum matching in G. Explain why this does not contradict the following theorem that we proved in class:

   Let B be a blossom with respect to M. Then M is a maximum size matching in G if and only if M/B is a maximum size matching in G/B.

3. Let $G = (V, E)$ be any graph. Let $S \subseteq V$ be any set of vertices such that there exists a matching M for which S is a subset of the matched vertices in M. Prove that there exists a *maximum* matching M^* for which all the vertices of S are matched as well.

4. Let $G = (V, E)$ be any graph and let $U \subseteq V$ be a set of vertices such that there exists a matching M of size $\frac{1}{2}(|U| + |V| - o(G\backslash U))$. Let $K_1, \ldots, K_l$ be the connected components of $G\backslash U$.

   (a) If M is *any* maximum matching, then M contains exactly $\left\lfloor \frac{|K_i|}{2} \right\rfloor$ edges from the subgraph of G induced by the vertices of K_i.

   (b) If M is *any* maximum matching, then each vertex $u \in U$ is matched to a vertex v in an odd component K_i of $G\backslash U$.

   (c) If M is *any* maximum matching, then the only exposed vertices must be in odd components of $G\backslash U$.

5. Consider the Tutte-Berge formula $\max_M |M| = \min_{U \subseteq V} \frac{1}{2}(|U| + |V| - o(G\backslash U))$. Can there exist different sets $U \subseteq V$ that minimize the right handside? If so, find an example; otherwise, prove that the set U that minimizes the right handside is unique.