Deceptively Uninspiring Homework 5

Due Wednesday May 21th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. Given a positive integer n, a finite sequence of positive integers $(a_1, \ldots, a_k)$ is said to be *coherent* if $\sum_{i=1}^{k} a_i = n$. Show that the number of coherent sequences of n is 2^{n-1}. For instance, here are the $4 = 2^{3-1}$ coherent sequences of 3: (3), $(2,1)$, $(1,2)$ and $(1,1,1)$.

2. Prove using induction that $n! > n^2$ for $n \geq 4$.

3. In a *labeled* tournament, the vertices are numbered from 1 to n. The labeling matters, but which vertex is drawn where doesn't matter. Find (and prove) a formula for the number of labeled tournaments with n vertices.

4. For $k \geq 3$, a k-cycle in a tournament is a sequence of vertices v_1, v_2, …, v_k with arcs $v_1 \to v_2 \to \ldots \to v_k \to v_1$. Prove that if a tournament contains a k-cycle for some $k > 3$, then it also contains a 3-cycle.