Deceptively Uninspiring Homework 4

Due Wednesday April 26th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. Give an example of a set S that contains an element x such that $x \in S$ and $x \subseteq S$.

2. Let A and B be sets. Prove that $A \setminus (A \cap B) = A \setminus B$.

3. Let A and B be sets. Prove that $A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$.

4. Let A, B, and C be sets. Prove that if $A \cup C \subseteq B \cup C$, then $A \setminus C \subseteq B$.

5. Let A and B be sets. Prove each of the following.
   (a) $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.
   (b) There exist sets A and B such that $\mathcal{P}(A \cup B) \nsubseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

6. List all equivalence relations on $\{a, b, c\}$. How many are there? How many relations are there on $\{a, b, c\}$?

7. Determine whether each of the following relations on $\mathbb{Z}$ is a partial ordering. Prove all your answers.
   (a) $R = \{(a, b) : |a - 1| \leq |b - 1|\}$
   (b) $R = \{(a, b) : a^2 \leq b^2\}$
   (c) $R = \{(a, b) : 2a < b\}$

8. Suppose A is a nonempty set and R is a relation with the property that, for all $a \in A$, there exists $b \in A$ such that aRb. Is R an equivalence relation on A? If yes, prove it; otherwise, state explicitly what fails.