

Deceptively Uninspiring Homework 3

Due Wednesday April 12th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like \LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. Suppose a and b are integers. Prove that $ab + a + b$ is even if and only if both a and b are even.
2. Suppose n is an integer. Prove that n^2 is even if and only if $4 \mid n^2$.
3. Show that $6 \mid n(n+1)(2n+1)$ for every positive integer n .
4. Show that $\sqrt{3}$ is irrational.
5. If x is irrational, show that $x + \frac{a}{b}$ is irrational for all $a, b \in \mathbb{Z} \setminus \{0\}$.
6. If x is irrational, show that $x \cdot \frac{a}{b}$ is irrational for all $a, b \in \mathbb{Z} \setminus \{0\}$.