Annie's Survival Kit 8 - Math 324

1. (10 points) Let S be the part of the surface $x^2+y^2+(z-1)^2 = 4$ (oriented with $\hat{\mathbf{n}}$ pointing outwards) lying above the plane $z = 1$. Let $\mathbf{F} = \langle 4x^3 + x - \sin(yz), -12x^2y + e^{z^3}, x^2 + y^2 \rangle$. Find the flux through S using the divergence theorem. Recall that the divergence theorem states that $\int\int_S \mathbf{F} \cdot \hat{\mathbf{n}} dS = \int\int\int_R \operatorname{div}(\mathbf{F}) dV$ when $\mathbf{F}$ is defined and differentiable everywhere on S and R where S is a closed surface and R is the region that it contains. Note here that S is not closed.

2. (10 points) Find the volume of the (open at both ends) half-donut parametrized by

$$\mathbf{r}(u, v) = \langle (3 + \cos(v)) \cos(u), (3 + \cos(v)) \sin(u), \sin(v) \rangle$$

for $u \in [0, \pi]$ and $v \in [0, 2\pi]$. (Hint: use the divergence theorem and an appropriate vector field.)

3. (10 points) Let S be a sphere of radius a centered at the origin. Calculate $\int\int_S x^2 - z \, dS$ by applying the divergence theorem.