Annie's Survival Kit 8 - Math 324
Double the fun!

1. (10 points) (a) (4 points) Let $\mathbf{F} = \langle x, y, z \rangle$ and let S be the part of the surface $z = \sqrt{x^2 + y^2}$ lying underneath the plane $z = 1$, where $\hat{\mathbf{n}}$ is pointing generally upwards/inwards. Draw S and a few vectors for $\hat{\mathbf{n}}$ and $\mathbf{F}$.

   (b) (6 points) Find $\int\int_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS$ either by parametrizing S or in any other way. Make sure every part of your answer is clear.

2. (10 points) (a) (8 points) Find the moment of inertia around the z-axis for the surface $x^2 + y^2 = 4$ with $0 \leq z \leq 1$ and with density equal to the square of the distance to the z-axis.

   (b) (2 points) Without doing further calculations, determine whether or not the moment of inertia around the z-axis for the surface $x^2 + y^2 = 4$ with $1 \leq z \leq 2$ is the same as in part (a). Explain your answer.

3. (10 points) Find the flux through the surface $x^2 + y^2 + (z - 1)^2 = 1$ oriented outwards/upwards for $\mathbf{F} = \langle -x, -y, -z \rangle$.

4. (10 points) Let S be the part of the surface $x^2+y^2+(z-1)^2 = 4$ (oriented with $\hat{\mathbf{n}}$ pointing outwards) lying above the plane $z = 1$. Let $\mathbf{F} = \langle 4x^3 + x - \sin(yz), -12x^2y + e^{z^3}, x^2 + y^2 \rangle$. Find the flux through S using the divergence theorem. Recall that the divergence theorem states that $\int\int_S \mathbf{F} \cdot \hat{\mathbf{n}} dS = \int\int\int_R \operatorname{div}(\mathbf{F}) dV$ when $\mathbf{F}$ is defined and differentiable everywhere on S and R where S is a closed surface and R is the region that it contains. Note here that S is not closed.

5. (10 points) Find the volume of the (open at both ends) half-donut parametrized by

$$\mathbf{r}(u, v) = \langle (3 + \cos(v)) \cos(u), (3 + \cos(v)) \sin(u), \sin(v) \rangle$$

   for $u \in [0, \pi]$ and $v \in [0, 2\pi]$. (Hint: use the divergence theorem and an appropriate vector field.)

6. (10 points) Let S be a sphere of radius a centered at the origin. Calculate $\int\int_S x^2 - z\, dS$ by applying the divergence theorem.