

Annie's Survival Kit 2 - Math 324

1. (10 points) (a) (7 points) Switch the order of integration of $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2+1}}^1 1 \, dz \, dx \, dy$ to $dr \, d\theta \, dz$.
- (b) (3 points) Knowing that $\int \int \int_R 1 \, dV$ calculates the volume of a region R , solve the previous triple integral without doing any calculations.
2. (10 points) Switch the order of integration of $\int_0^\pi \int_0^1 \int_{-1}^1 zr^3 \, dz \, dr \, d\theta$ to $dy \, dx \, dz$.
3. (10 points) Consider a solid cone of height $\sqrt{3}$ with a 120° vertex angle. Its density at point P is equal to the distance from P to the central axis of the cone. Set up the integrals for the mass of the cone using cylindrical coordinates in two different orders: $dz \, dr \, d\theta$ and $dr \, d\theta \, dz$. Do not evaluate those integrals.

Hints:

- Choose and place the coordinate system to get the easiest integral possible.
- The mass of a solid region R with density δ is $\int \int \int_R \delta \, dV$.
- If the cone has a $\frac{2\pi}{3}$ vertex angle (the angle between its sides), what is the slope of its sides? How does the slope fit into the equation for a cone?