Quiz 4 Name ___________________

1. Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ form an orthonormal basis (meaning it's an orthogonal basis and $\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{w}\| = 1$), and $\mathbf{x} \in \mathbb{R}^3$ with $\mathbf{x} = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$. Write a, b and c in terms of $\mathbf{x}, \mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.

Solution:
Since $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are orthogonal, we have a formula for their coefficients, which simplifies since $\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{w}\| = 1$. We get the following:
$a = \frac{\mathbf{x}\cdot\mathbf{u}}{\|\mathbf{u}\|^2} = \mathbf{x}\cdot\mathbf{u}$
$b = \frac{\mathbf{x}\cdot\mathbf{v}}{\|\mathbf{v}\|^2} = \mathbf{x}\cdot\mathbf{v}$
$c = \frac{\mathbf{x}\cdot\mathbf{w}}{\|\mathbf{w}\|^2} = \mathbf{x}\cdot\mathbf{w}$

A lot of people said that this was the projection of $\mathbf{x}$ onto $\mathbf{u}$, which is close but not quite the same thing - we defined the projection to be a vector, for example:
$\text{proj}_{\mathbf{u}}\mathbf{x} = \frac{\mathbf{x}\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u} = a\mathbf{u}$.

2. Let $A = \begin{bmatrix} 0 & 1 \\ 2 & 7 \end{bmatrix}$. For the following values of λ, determine if λ is an eigenvalue for A, and if so find a basis for the corresponding eigenspace. (Note: you must show your work to receive full credit.)

(a) $\lambda = 0$

(b) $\lambda = 2$

(a) Solution: $\det(A) = -2 \neq 0$, so 0 is not an eigenvalue of A.

(b) Solution: $\det(A - 2I) = -12 \neq 0$, so 2 is not an eigenvalue of A.

Remark: A lot of people apparently computed that 2 was not an eigenvalue, but then they second guessed themselves and found an eigenspace anyway. Try to be really clear about what you are doing and why things are the way they are, so that you can be confident about your answers.