

Participation 5 (Take-home quiz due Nov 9)

Problem 1. Suppose A is a nonsingular (3×3) matrix with $A^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 1 \\ 3 & -8 & 4 \end{bmatrix}$.

(a) If $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, find \mathbf{x} .

$$\text{We have } \mathbf{x} = I\mathbf{x} = A^{-1}A\mathbf{x} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 1 \\ 3 & -8 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ -1 \end{bmatrix}.$$

Note: there was another way to solve this problem, and some people were able to do this successfully. However, it had a lot of computation and most people who tried that made computation errors. I really want to encourage you to:

- Use the information you have available. (In this problem you knew A^{-1} rather than A . Think about what you can do with that.)
- Use the tools you have learned. (Ask yourself: what have we learned about inverses?)
- Think about how you are going to approach problems before you attack. Even if you already see one way to do a problem, ask yourself if you have some knowledge or tools that will make something easier. (Theorems can be really fantastic tools!)

(b) Suppose B is another nonsingular matrix with $B^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find the inverse of the matrix AB .

$$\text{Solution: } (AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 1 \\ 3 & -8 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -5 & 5 \\ 0 & 2 & 9 \\ 3 & -8 & 4 \end{bmatrix}.$$

- Again, there was another way to do this problem, but it was much harder than this way. Look to use the information and tools you have available. If I give another problem like this and you choose to do it the hard way, I will not be particularly lenient on calculation errors.
- If you messed up the matrix multiplication, please review this and do some practice matrix multiplication. If you're struggling with this, come talk to me. You should be fairly good at matrix multiplication by now.
- If you thought that A^{-1} and B^{-1} would appear in the other order, remind yourselves why they reverse order. It is not difficult to prove that $(AB)^{-1} = B^{-1}A^{-1}$. (Even on a test, you could reprove it to yourself if you were not sure.)
- Make sure if you are referring to the inverse of AB that you use parentheses to write $(AB)^{-1}$. Otherwise it is unclear what you mean.