Math 308O
Winter 2015
Midterm 2
February 25, 2015
Time Limit: 50 Minutes

Name (Print): ____________________

Instructor ____________________

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use a scientific calculator and a sheet of notes, 8.5×11 and handwritten by you, but no other devices, books, or notes are permitted.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. I can also provide paper to attach to your exam if needed.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Let $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 4 & 4 & 7 & 2 \\ 1 & 3 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

   (a) (5 points) Compute the rank of A.

   (b) (3 points) Compute the nullity of A.

   (c) (2 points) Is A invertible? Explain.

2. Read each of the following statements carefully, and decide whether it is true or false. You are not required to justify your answers, but I recommend justifying them to yourself.

   (a) (2 points) Suppose $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation. If $m < n$, then T is one-to-one.

   (b) (2 points) Suppose A is an $(n \times n)$ matrix with linearly independent columns, and define $T(\mathbf{x}) = \mathrm{A}\mathbf{x}$. Then T is onto.

   (c) (2 points) Suppose A and B are nonsingular $(n \times n)$ matrices. Then $(AB)^2 = A^2B^2$.

   (d) (2 points) If A is an $(n \times n)$ matrix with linearly independent column vectors, the row vectors of A are also linearly independent.

   (e) (2 points) True or false: If A is an $(n \times n)$ matrix, then $A + A^T$ is symmetric.

3. Suppose A is a nonsingular (3×3) matrix with $A^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 1 \\ 3 & -8 & 4 \end{bmatrix}$.

(a) (5 points) If $A^T x = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, find x. Note that you do not need to compute A to do this calculation.

(b) (5 points) Suppose B is another nonsingular matrix with $B^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Compute the matrix $(AB)^{-1}$.

4. (10 points) Choose **one** of the following statements to prove. Please note you will not get additional credit for doing some of each. Make it **VERY CLEAR** which proof you want me to grade.

   (a) Suppose that A is an $(n \times m)$ matrix. Prove that the null space of A is a subspace of $\mathbb{R}^m$.

   (b) Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is an invertible linear transformation. Prove that T^{-1} is also a linear transformation.