Math 308O
Winter 2015
Midterm 1
January 28, 2015
Time Limit: 50 Minutes

Name (Print): ____________________

Instructor ____________________

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use a scientific calculator and a sheet of notes, 8.5×11 and handwritten by you, but no other devices, books, or notes are permitted.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. I can also provide paper to attach to your exam if needed.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Suppose we know that a polynomial takes the form $f(x) = ax^2 + bx + c$, and goes through the points $(1, 1), (2, 1)$, and $(-1, -5)$. The goal of this problem will be to find a, b and c.

   (a) (3 points) Set up a system of linear equations and find the corresponding augmented matrix.

   (b) (4 points) Put the augmented matrix into echelon form.

   (c) (3 points) Solve for a, b and c. Please check your work.

2. Read each of the following statements carefully, and decide whether it is true or false. You are not required to justify your answers, but I recommend justifying them to yourself.

   (a) (2 points) A homogeneous linear system with n variables and n equations will have exactly one solution.

   (b) (2 points) Suppose $\mathbf{x_1}$ and $\mathbf{x_2}$ are vectors in $\mathbb{R}^m$, $\mathbf{b}$ is a vector in $\mathbb{R}^n$, and A is an $n \times m$ matrix. If $\mathbf{x_1}$ and $\mathbf{x_2}$ are both solutions to the equation $A\mathbf{x} = \mathbf{b}$, then so is the vector $\mathbf{x_1} + \mathbf{x_2}$.

   (c) (2 points) The set $\left\{ \begin{bmatrix} 2 \\ 3 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 15 \\ 0 \\ 0 \end{bmatrix} \right\}$ is linearly dependent.

   (d) (2 points) If the columns of A are linearly dependent, then the equation $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

   (e) (2 points) If the set $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent in $\mathbb{R}^3$, then the set $\{\mathbf{0}, \mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ spans $\mathbb{R}^3$.

3. (a) (3 points) Find vectors $\mathbf{u_1}, \mathbf{u_2}$ and $\mathbf{u_3}$ in $\mathbb{R}^3$ so that the sets $\{\mathbf{u_1}, \mathbf{u_2}\}, \{\mathbf{u_2}, \mathbf{u_3}\}$ and $\{\mathbf{u_1}, \mathbf{u_3}\}$ are linearly independent, but the set $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly dependent.

(b) (3 points) Find five distinct nonzero vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}, \mathbf{v_5}$ in $\mathbb{R}^3$ so that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}, \mathbf{v_5}\}$ does not span $\mathbb{R}^3$.

(c) (4 points) Find a value for k so that the set $\left\{ \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ k \end{bmatrix} \right\}$ is linearly dependent.

4. (10 points) Give a geometric interpretation (including the exact equation) for

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \right\}.$$