Lecture #15, Wednesday, May 19, 1999

Handouts: $TR$ from $MR$.

Agenda: Review of SWS #10, WS #20 & #21
Announcements

- Friday’s Quiz:
  Covers WS #20 & 21 and the material covered in quiz and lecture from Monday through Thursday.

- NOTEBOOKS: Notebooks will be collected next Monday, May 24, between 2:45 and 4PM or Tuesday, May 25, between 12:00 Noon and 4:00PM in my office - Padelford C-337. Work through WS #22 will be graded. If you wish to wait when you hand in your notebook, it will be graded and immediately returned to you.
Rate of Ascent of Balloon

(quick review of Monday’s worksheet)

A hot-air balloon is launched at 12:00 noon. Let \( y = h(t) \) denote the height (in feet) of the balloon above the ground at time \( t \) (in minutes) after 12:00 noon. Notice that \( h(0) = 0 \).

Key Question: How many feet is the balloon above the ground at 12:30 pm (when \( t = 30 \))?

Hopefully by now you have answered this question by graphing the height function \( y = h(t) \) (we got through this graph up to \( t = 20 \) on Monday).
Rate of Ascent $r(t)$:
Key Observation

The change

$$h(b) - h(a)$$

in the height of the balloon between $t = a$ and a later time, $t = b$ is the signed area under the part of the rate of ascent graph between $t = a$ and $t = b$. 
The rate of ascent is constant (15 ft/min), so

\[ h(t) - h(0) = 15 \cdot (t - 0) = 15t \]

Since \( h(0) = 0 \), get the formula

\[ h(t) = 15t \text{ for } 0 \leq t \leq 10. \]

So \( h(10) = 150 \text{ ft.} \)
The rate changes linearly, so we can use the formula for the area of a trapezoid to compute changes in height:

$$h(t) - h(10) = \frac{r(10) + r(t)}{2} \cdot (t - 10)$$

or

$$h(t) = h(10) + \frac{r(10) + r(t)}{2} \cdot (t - 10)$$

So

$$h(15) = 150 + \frac{(-5+10)}{2} \cdot 5 = 112.5 \text{ ft}$$

$$h(20) = 150 + \frac{(-5+15)}{2} \cdot 10 = 50 \text{ ft}$$

Note that each time you use the area of a trapezoid in this problem you are actually saying ”Change in height is average rate of ascent times time interval(with that average rate)”. This is just a version of ”Distance travelled equals average speed times time of travel”.

Computation for $10 \leq t \leq 20$
\textbf{Computation for }20 \leq t \leq 25\textbf{ }

The graph of \( r(t) \) is curved, so areas are not given exactly by the “trapezoid formula”. But it DOES give an approximation to the correct area. For instance

\[
h(25) \approx h(20) + \frac{r(20) + r(25)}{2} \cdot (25 - 20) = -18.75
\]

\[
h(30) \approx h(25) + \frac{r(25) + r(30)}{2} \cdot (30 - 25) = -62.5
\]
Graph of $h(t)$

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(t)$</td>
<td>0</td>
<td>75</td>
<td>150</td>
<td>112</td>
<td>50</td>
<td>-18.75</td>
<td>-62.5</td>
</tr>
</tbody>
</table>
Handout: Getting TR from MR

**Idea:** Apply a similar procedure to compute Total Revenue (TR) from Marginal Revenue (MR).

**Setup:**

1. **Top:** Marginal Revenue \( MR = r(q) \)
   - Horizontal axis: \( q = 1, 2, \ldots, 15 \) thous. of items
   - Vertical axis: \( w = 1, 2, \ldots, 7 \) dollars/item

2. **Bottom:** Total Revenue \( TR = R(q) \)
   - Horizontal axis: \( q = 1, 2, \ldots, 15 \) thous. of items
   - Vertical axis: \( w = 5, 10, \ldots, 70 \) thous. of dollars
How to get TR from MR

- $q = 1$ to $q = 4$: each item adds $1$. So $TR(q) = q$. e.g. $TR(4) = 4$ thousand dollars

- $q = 4$ to $q = 6$: first items add less, later items add more, so graph curves up.

  Average $MR$ is $3.50$ so $TR(6) - TR(4) = 3.50 \times 2 = 7$ thousand dollars.
  
  $TR(6) = 11$ thousand

- $q = 6$ to $q = 9$: each item adds $6$. So $TR(q) = TR(6) + 6(q - 6)$ thousand dollars.
  
  $TR(9) = 29$ thousand

- $q > 9$ early items add more, later items add less, so curves down.

  Can use trapezoids to approximate area.
Graph of $R(q)$

<table>
<thead>
<tr>
<th>$q$</th>
<th>0</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(q)$</td>
<td>0</td>
<td>4</td>
<td>11</td>
<td>23</td>
<td>29</td>
<td>41</td>
<td>51</td>
</tr>
</tbody>
</table>
\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
q & 0 & 4 & 6 & 8 & 9 & 11 & 13 \\
\hline
R(q) & & & & & & & \\
\hline
\end{array}
\]