Lecture #13, Wednesday, May 12, 1999

**Handouts:** Seating chart for Friday MidTerm Exam.

**Agenda:** Pre-Test #2
Announcements

- MIDTERM #2 is Friday
  THERE IS A SEATING CHART! Find your row TODAY, so Friday can proceed quickly.

- STUDY PROBLEMS: Pre-test #2 has four types of problems. These will correspond to the 4 problems on Friday’s Mid-term; in particular there will be a constrained optimization problem. In addition to the pre-test problems, you are expected to work all of study problems-Part II, #33-#43. (Warning none of these “exam-like” study problems covers constrained optimization.)
We wish to optimize the function

\[ P = L^{1/3}C^{2/3} \]

subject to the constraint

\[ 100L + 150C = 30,000. \]

(The variable \( L \) stands for “labor” and the variable \( C \) stands for “capital.” This function for \( P \) in terms of \( L \) and \( C \) is called a production function.)

a) Write the variable \( P \) as a function of the single variable \( L \) by combining the objective and constraint functions.

b) Using your answer to (a) find the value of \( L \) that maximizes \( P \). [Note that after you have set the derivative equal to 0, you can simplify things by multiplying the equation by a carefully chosen power of \( L \).]

c) What are the values of \( C \) and \( P \) that correspond to the value of \( L \) you found in (b)?
Pre-Test Question 9

The Demand Curve for Blivets shown in your text has the formula

\[ h(q) = 5 - \sqrt{q + 3}. \]

We get Total Revenue from this curve by the recipe \( R(q) = q \cdot h(q) \).

a) Write the formulas for \( R(q) \) and \( R'(q) \).

b) Find the positive (non-zero) value of \( q \) at which \( R(q) = 0 \).

c) Find the value of \( q \) in the interval between \( q = 0 \) and the answer you gave in (b) at which \( R(q) \) reaches its largest value.

d) The Total Cost of manufacturing Blivets is given by the formula \( C(q) = q + 6 \). Find the quantity of Blivets to be produced to maximize profits.
Interpreting Derivatives 1

Consider the following table of values of a function $z = f(x, y)$.

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Slide 5
a) Sketch the graph of the function 
\[ z = h(t) = f(t, 6) \] for values between 
\[ t = 0 \text{ and } t = 6. \]

b) Sketch the graph of the function 
\[ z = k(t) = f(4, t) \] for values between 
\[ t = 0 \text{ and } t = 12. \]

c) Using the graphs that you drew, estimate the following 
\[ \frac{\partial f(4, 6)}{\partial x} \quad \frac{\partial f(4, 6)}{\partial y} \]
You know the following information about the function \( w = H(u, v) \)

\[
H(2, 3) = -1, \quad \frac{\partial H(2, 3)}{\partial u} = 0.5 \quad \frac{\partial H(2, 3)}{\partial v} = -2
\]

\[
\frac{\partial^2 H(2, 3)}{\partial u^2} < 0, \quad \frac{\partial^2 H(2, 3)}{\partial v^2} > 0.
\]

(i) Use this information to sketch [as best you can] the graph of the function

\[
k(t) = H(t, 3)
\]

for \( t \) near \( t = 2 \).

(ii) Use this information to sketch [as best you can] the graph of the function

\[
f(t) = H(2, t)
\]

for \( t \) near \( t = 3 \).
Interpreting Partial Derivatives 3

You know the following information about the function \( z = L(s, t) \)

\[
L(2, 3) = -1, \quad \frac{\partial L(2, 3)}{\partial s} = 0, \quad \frac{\partial L(2, 3)}{\partial t} = 0
\]

\[
\frac{\partial^2 L(2, 3)}{\partial s^2} = -1, \quad \frac{\partial^2 L(2, 3)}{\partial t^2} = -2, \quad \frac{\partial^2 L(2, 3)}{\partial s \partial t} = 1.5.
\]

What can you say about the point \((s, t) = (2, 3)\)?