Handouts: 2nd Derivative drill.

Notebooks: Pick up from alphabetical piles on Lecture Table.

Suggestion Box: Remember e-mail to "m112a-box@ms.washington.edu" with Subject: #######(your student id) will be forwarded (anonymously) to me. Use it to send any suggestions, criticisms, (anonymous) praise, Questions for the Q&A on Friday. (If you wish to be anonymous DO NOT add any signature to your e-mail - none of the "content" is stripped away by the "suggestion box" forwarding software.

Agenda: Introduction to WS #16: Second derivatives; “Percent Rate of Change".
Announcements

- Friday’s Quiz covers WS #14–#16.
Types of Optimization Problems:

- **Type 1** — Find optima \textit{without} an interval “local optima”
- **Type 2** — find optima \textit{with} an interval.

“Find the optima of \( f(x) \)” means find all \( x \) where \( f(x) \) has a local maximum and where \( f(x) \) has a local minimum.

“Find the optima of \( f(x) \) in the interval from \( x = a \) to \( x = b \)” means: find the \( x \) in the interval for which \( f(x) \) is highest or lowest among all values of \( x \) in the interval.
Strategy for finding global optima:

- **Step 1: Find the candidates:**
  - (1) The points between $a$ and $b$ where $f'(x) = 0$.
    (These are the points where the tangent is horizontal.)
  - (2) the endpoints of the interval:
    $x = a$ and $x = b$

- **Step 2: Check the candidates:**
  - Compute the value of $f$ for each candidate.
  - The global maximum is the candidate yielding the largest value of $f$.
  - The global minimum is the candidate yielding the smallest value of $f$. 
Example:

Find the local optima of

\[ g(x) = -2x^3 + 9x^2 - 12x + 5. \]

\[ g'(x) = -6x^2 + 18x - 12 \]
\[ = -6(x^2 - 3x + 2) = -6(x - 1)(x - 2) \]

\[ g'(x) = 0 \text{ for } x = 1 \text{ and } x = 2. \]

Notice that

- \( g'(x) < 0 \) for \( x \) to the left of \( x = 1 \).
- \( g'(x) > 0 \) for \( x \) a immediately to the right.

So \( x = 1 \) gives a local minimum.

Similarly

- \( g'(x) > 0 \) for \( x \) to the left of \( x = 2 \).
- \( g'(x) < 0 \) for \( x \) a immediately to the right.

So \( x = 2 \) gives a local maximum.
“Concavity”

1. What would the graph of \( y = f(x) \) look like for \( x \) near 5 if

\[ f(5) = 3 \quad f'(5) = 0 \quad f''(5) = 2 \]

First: what would the graph of
\( y' = f'(x) \) look like for \( x \) near 5?
From this: what would the graph of
\( y = f(x) \) look like for \( x \) near 5?
ANS: \( \cup \)-shaped

2. Same question, but with
\[ f(5) = 3 \quad f'(5) = 0 \quad f''(5) = -2 \]
ANS: \( \cap \)-shaped

3. Same question, but with
\[ f(5) = 3 \quad f'(5) = 0 \quad f''(5) = 0 \]
ANS: NOT ENOUGH INFORMATION.
The Second Derivative Test

If \( f'(r) = 0 \), then

A. If \( f''(r) < 0 \), then \( f(x) \) reaches a local maximum at \( x = r \).

B. If \( f''(r) > 0 \), then \( f(x) \) reaches a local minimum at \( x = r \).

C. If \( f''(r) = 0 \), then the test does not tell us anything.
Example:

Find the local optima of

\[ g(x) = -2x^3 + 9x^2 - 12x + 5. \]

\[ g'(x) = -6x^2 + 18x - 12 \]
\[ = -6(x^2 - 3x + 2) \]
\[ = -6(x - 1)(x - 2) \]

\[ g''(x) = -12x + 18 \]
Slide 9
(1) On the grid to the right, sketch the graph of \( y = f(x) \) for \( x \) near 5 if
\[ f(5) = 3, \quad f'(5) = 1, \quad f''(5) = 2? \]

(2) Sketch the graph of \( y = f(x) \) for \( x \) near -5 if
\[ f(-5) = 6, \quad f'(-5) = 1, \quad f''(-5) = -2? \]

(3) Sketch the graph of \( y = f(x) \) for \( x \) near 5 if
\[ f(5) = -7, \quad f'(5) = -1, \quad f''(5) = 2? \]

(4) Sketch the graph of \( y = f(x) \) for \( x \) near -5 if
\[ f(-5) = -4, \quad f'(-5) = -1, \quad f''(-5) = -2? \]

(5) On the grid to the right, sketch the graph of \( y = f(x) \) for \( x \) near 2 if
\[ f(2) = 5, \quad f'(2) = 0, \quad f''(2) = 1? \]

(6) Sketch the graph of \( y = f(x) \) for \( x \) near 2 if
\[ f(2) = -5, \quad f'(2) = 0, \quad f''(2) = -1? \]

(7) Sketch the graph of \( y = f(x) \) for \( x \) near -7 if
\[ f(-7) = 5, \quad f'(-7) = 0, \quad f''(-7) = 0? \]

(8) Sketch the graph of \( y = f(x) \) for \( x \) near -7 if
\[ f(-7) = -5, \quad f'(-7) = 0, \quad f''(-7) = 0? \]