Lecture #4, Wednesday, April 7, 1999

Handout: List of derivative rules and examples to compute. (Not handed out - just showed slide.)

Announcements: None (Actually reminded class about CMU B006(004) and lots of hours for Study Sessions.)

Agenda:

- Mention Examples from WS #8 done in Quiz section
- Notation and first set of rules
- Intro to WS #10
Schedule for week:

- **Wednesday**: Review WS #8 and Intro to WS #9 & #10
  (Homework: finish WS #9 & #10.
- **Thursday Quiz**: Practice working with derivatives:
  SWS-03. Homework WS #11
- **Friday**: Q and A plus Quiz #2 (20 min).
- **Weekend**: Review WS #1–#11 and prepare for Pretest #1
Example from WS #8

You should have done formula work in Quiz section to deal with the following:

You are given

\[ B(t) = -t - \frac{25}{t + 1} + 25 \]

(Position (measured in ft) of Car B at time \( t \) (measured in mins)

1. What is the average speed of Car B between 2 minutes and 4 minutes?

2. What is the average speed of Car B over the first 4 minutes?
3. What is the formula for $B'(t)$, the instantaneous speed of the car?

4. How fast is Car B going after 2 minutes?

Graphs showing $B(t)$ and $B'(t)$ handed out Monday.
Other Symbols for the Derivative:

- Suppose $y = f(x)$. Then all of these mean the same thing:

$$f'(x) = \frac{dy}{dx} = y'$$

For instance, if we write $y = x^2 + 3x - 4$ then we write

$$\frac{dy}{dx} = 2x + 3$$

(Reviewed how to get this formula.) At other times, we even write

$$(x^2 + 3x - 4)' = 2x + 3,$$

meaning that $2x + 3$ is the derivative of the function defined by the formula

$y = x^2 + 3x - 4.$
Ideas behind Differentiation Rules

- How are slopes of $g(t)$ and $f(t) = c \cdot g(t)$ related? In general multiplying vertical measurements by $c$ will multiply the numerators in slope calculations by $c$. So we’ll get coefficient rule for derivatives.

\[
(c \cdot g(t))' = c \cdot g'(t)
\]

- How is slope of a sum related to slopes of each summand? Look at picture to see that changes in height on graph of $f(x) = g(x) + h(x)$ are just the sums of changes of height on $g(x)$ and $h(x)$ graphs. This leads to the sum rule.

\[
(g(x) + h(x))' = g'(x) + h'(x)
\]
Pattern for power rule

- Examples have shown:
  - derivative of linear is constant:
    
    \[(3x - 4)' = 3.\]

  - derivative of quadratics are linear:
    
    \[(x^2 + 3x - 4)' = (x^2)' + (3x - 4)' = 2x + 3.\]

  - derivative of cubic is a quadratic:
    
    \[(x^3 + x^2 + 3x - 4)' = 3x^2 + 2x + 3.\]

From this we can see \(x' = 1, (x^2)' = 2x, (x^3)' = 3x^2.\)

- What is the pattern here? Fill in question marks: \((x^4)' = ?, (x^{-1})' = ?.\) The pattern even works to get derivatives of fractional powers. \((x^{\frac{3}{2}})' = \frac{3}{2}(x^{\frac{3}{2}-1}) = \frac{3}{2}x^{\frac{1}{2}}.\)

- All of these fit the power rule:

  \[(x^n)' = nx^{n-1}.\]
First set of Differentiation Rules:

**POWER RULE:** If \( f(x) = x^n \) then
\[
f'(x) = nx^{n-1}.
\]

**COEFFICIENT RULE:** If \( f(x) = c \cdot g(x) \)
then \( f'(x) = cg'(x) \).

**SUM RULE:** If \( f(x) = g(x) + h(x) \) then
\[
f'(x) = g'(x) + h'(x)
\]

**Examples:**

1. \( g(x) = x^{3/2} \)
2. \( h(u) = 5u^2 \)
3. \( h(t) = \frac{1}{t} \)
4. \( h(x) = 5\sqrt{x} + \frac{7}{x} \)
5. \( f(y) = \sqrt{\pi x} \)
**Some Interpretations of the derivative.**

<table>
<thead>
<tr>
<th>$y = f(x)$</th>
<th>$y' = f'(x) = \frac{dy}{dx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td>Derived Graph (slope of tangent line)</td>
</tr>
<tr>
<td>$A(t) =$ altitude</td>
<td>$A'(t) =$ Rate of ascent</td>
</tr>
<tr>
<td>$V(t) =$ volume of water in a tank</td>
<td>$V'(t) =$ Rate that water is flowing into tank</td>
</tr>
<tr>
<td>$R(q) =$ Total Revenue</td>
<td>$R'(q) =$ Marginal Revenue</td>
</tr>
<tr>
<td>$C(q) =$ Total Cost</td>
<td>$C'(q) =$ Marginal Cost</td>
</tr>
<tr>
<td>$C'(q) =$ Total Variable Cost</td>
<td>$C'(q) =$ Marginal Cost</td>
</tr>
<tr>
<td>$P(q) =$ Net Profit $= R(q) - C(q)$</td>
<td>$P'(q) =$ Marginal Profit $= R'(q) - C'(q)$</td>
</tr>
</tbody>
</table>
Introduction to WS #10

Situation:

- $q =$ quantity of Framits (in hundreds of items)
- $R(q) =$ Total Revenue (TR) (in hundreds of dollars)

$$R(q) = -0.2q^2 + 2q$$

- $C(q) =$ Total Variable Cost (TVC) (in hundreds of dollars)

$$C(q) = \frac{1}{30}q^3 - \frac{3}{10}q^2 + q$$

- Total Cost (TC) is given by the formula

$$TC = TVC + \text{Fixed Cost}$$
TR: \( R(q) = -0.2q^2 + 2q \)

TVC: \( C(q) = \frac{1}{30}q^3 - \frac{3}{10}q^2 + q \)
Questions about TR

- Suppose sales increase from 200 Framits to 600 Framits. On average, how much does each of these 400 Framits add to TR?
  - Ruler and pencil: slope = \( \frac{1.6}{4} = \$0.40 \).
  - Formula:
    \[
    \frac{TR(6) - TR(2)}{4} = \frac{4.8 - 3.2}{4} = 0.4
    \]
- \( MR \) for \( q = 2 \) ( \(-0.4q + 2 = 1.20 \ $/item\) )
- \( MR \) for \( q = 6 \) ( \(-0.40 \ $/item\) )
- For which quantity \( q \) is it true that selling one more item does not change the Total Revenue?
  [Want \( MR(q) = 0 \) or \(-0.4q + 2 = 0 \) or \( q = 5 \) hundred items.]
Note: Showed slides up to here on Wednesday. Didn’t discuss much of this slide and didn’t look at the remaining ones at all. They’re useful and should be considered before Friday quiz.


Questions about TC and TVC

- How much does the 101st framit add to the TC?
- How much does the 101st framit add to the TR?
- How much does the 101st framit add to the Profit?
- How many Framits should we sell if we want to maximize our profit?

Want \( MR = MC \)

\[-0.4q + 2 = 1 - \frac{3}{5}q + \frac{q^2}{10}.
\]

Or

\[-\frac{q^2}{10} + 0.2q + 1 = 0\]

\( q = -2.31662, 4.31662. \) Only \( q = 4.31662 \) makes sense. So get max profit if we sell 432 items (to nearest whole).
Questions bout Profit

Another approach: Profit = $TR - TC$:

$$P(q) = R(q) - C(q) - 0.8$$
$$= -\frac{q^3}{30} + 0.1q^2 + q - 0.8$$

$$MP = P'(q) = -\frac{q^2}{10} + 0.2q + 1$$

When is $MP = 0$? Use quadratic formula

$q = 4.3166$. 