I strongly recommend that you read Chapter 8 (you may know all or some of it depending on your background in geometry).

Written assignment (4 problems).

Writing Problem 1. Let ABC be a triangle, and A', B' and C' be points on BC, AC and AB respectively such that AA', BB' and CC' are concurrent at the point G. Prove that

$$\frac{GA'}{AA'} + \frac{GB'}{BB'} + \frac{GC'}{CC'} = 1$$

Writing Problem 2. The triangle formed by joining midpoints of the sides of a given triangle is called the medial triangle.

1. Prove that the medial triangle and the original triangle have the same centroid
2. Prove that the orthocenter of the medial triangle is the center of the circumscribed circle of the original triangle.

Writing Problem 3. Let $\triangle ABC$ be an acute triangle. Prove that

$$h_a > \frac{1}{2}(AC + AB - BC)$$

where h_a is the length of the altitude on the side BC.

Writing Problem 4. Let C be a circle in the xy-plane with center on the y-axis and passing through points $A = (0, a)$, $B = (0, b)$ with $0 < a < b$. Let O be the origin $(0, 0)$, let P be any point on the circle, and let Q be the intersection of the line through P and A with the x-axis. Prove that

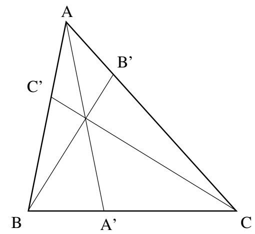
$$\angle BQP \cong \angle BOP.$$

<u>EC.</u> In a triangle ABC choose point A_1 on side BC, point B_1 on side CA, and point C_1 on side AB in such a way that the three segments AA_1, BB_1, and CC_1 intersect in one point P. Prove that P is the centroid of triangle ABC if and only if P is the centroid of triangle $A_1B_1C_1$.

Presentation assignment (4 problems).

Presentation Problem 1. Prove *Ceva's theorem*: Let ABC be a triangle, and A', B' and C' be points on the interiors of BC, AC and AB respectively. Then AA', BB' and CC' are concurrent if and only if $\frac{AC' \cdot BA' \cdot CB'}{C'B \cdot A'C \cdot B'A} = 1$.

Note: for the forward direction you **must** use an argument different from the one presented in the book. For example, an argument with weights. This is reviewed in 8.4.21 in Zeitz. Each part of this problem will be worth 3 presentation points.



Presentation Problem 2. Prove Pick's formula: Let P be a convex polygon with vertices on the integer lattice. Let i be the number of the interior integer points of P, and b be the number of integer points on the sides (boundary) of the polygon. Prove that the area of P equals $i + \frac{b}{2} - 1$.

Presentation Problem 3. Let S be a circle with the center (a, b) where a, b are both irrational numbers. Show that S cannot contain more than 2 points with rational coordinates.

Presentation Problem 4. Find

$$\arctan\frac{1}{2} + \arctan\frac{1}{8} + \ldots + \arctan\frac{1}{2n^2} \quad .$$

Arctan(t) for $t \geq 0$ denotes the number θ in the interval $0 \leq \theta < \pi/2$ with $\tan(\theta) = t$. You might find the following formula useful: $\arctan x - \arctan y = \arctan \frac{x-y}{1+xy}$.