Math 380a *HOMEWORK #9* *due Tue, December 6, 2016*

Written assignment (4 problems).

W1. From where he stands, one step toward the cliff would send a drunken man over the edge. He takes random steps on a line, either toward or away from the cliff. At any step his probability of taking a step toward the cliff is p (the probability of stepping away from the cliff is $1-p$).

a) What is the probability that his "drunken walk" will eventually take him off the cliff?

b) What is the probability that he falls off the cliff after precisely k steps?

W2. Let a be the diameter of a small coin. Given a chessboard C made of squares of size $d \times d$ and with delimiting lines of negligible width, throw the coin "at random" on the chessboard. By "at random" we mean that the center of the circular coin can fall uniformly on any point on the chessboard (the margin of the coin may be outside the chessboard, if the center is close to the edge). What is the probability that the coin will fall entirely within one of the individual $d \times d$ squares?

W3. Suppose n numbers are chosen uniformly and randomly from the interval $[0, 1]$. What is the expected value of the largest of them?

W4. Let C be the circle of radius 1 centered at the origin. Choose a point P at random on the circle, and a point Q at random inside the circle. What is the probability that the rectangle with diagonal PQ and sides parallel to the axes lies entirely in or on the circle?

Extra Credit Problem 1. Suppose you have a biased coin c, with *unknown* bias p. Give a procedure which terminates in a finite number of steps, and which can be used to simulate a fair coin by just flipping c (and recording the outcomes).

Presentation assignment (5 problems)

P1. Let p_n be the probability that two numbers selected at random, independently and uniformly, from $\{1, 2, \ldots, n\}$ sum up to a square (e.g., $1 + 3 = 2^2$). Find $\lim_{n\to\infty} p_n\sqrt{n}$.

P2. Suppose that x and y are chosen at random (with uniform density) and independently from the interval $(0, 1)$. What is the probability that the closest integer to x/y is even?

P3. Suppose you are given a set of n *biased* coins, such that the probability that the mth coin will land on "heads" is $\frac{1}{2m+1}$. If you flip all n coins independently exactly once, what is the probability that you get an odd number of heads? (Hint: write a double recurrence for the probability of getting an odd/even number of heads.)

P4. Ioana and Julia are tossing a coin. Ioana made 2016 tosses, and Julia—2017. What is the probability that Julia got more heads than Ioana?