Math 380 **HOMEWORK #8** *due Tue, November 24, 2015*

Written assignment (3 problems).

W1. Compute the exponential generating function for the sequence satisfying the recurrence $a_0 = a_1 = 1$, $a_n = a_{n-1} + (n-1)a_{n-2}$. Then use the generating function to obtain an expression for a_n (in the form of a sum).

W2. Prove Vandermonde formula using generating functions:

$$\sum_{j=0}^{k} \binom{n}{j}\binom{m}{k-j} = \binom{n+m}{k}$$

W3.

a) Let $f(x)$ be the ordinary generating function for the sequence $\{a_n\}_{n\geq 0}$. Show that $a_n = f^{(n)}(0)/n!$, where $f^{(n)}$ is the nth formal derivative of f.

b) Fix an integer $k \geq 1$. Let a_n be the number of solutions of the equation $x_1 + x_2 + \ldots + x_k = n$ where $x_1, \ldots, x_k$ are non-negative integers. Find a formula for a_n by using generating functions.

Presentation assignment (4 problems).

P1. Find and prove the closed form for Fibonacci numbers.

P2. Using generating functions, prove the fact we already used in one of the presentation problems: any integer weight between 1 and 100 can be balanced uniquely on a two cup scale using the weights 1, 3, 9, 27 and 81.

P3. Prove that F_{238} is divisible by 239. No computer programs or explicit calculations, please!

P4. Prove that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, for all $n \geq 1$, via ordinary generating functions, as follows. First, compute the ordinary generating function for $a_n = n^2$. Then, multiply it to another well-known ordinary generating function, to obtain the ordinary generating function for $\{\sum_{k=1}^{n} k^2\}_{n\geq 1}$. Finally, use the form of the obtained product to deduce the formula for the sum of squares.