Math 380a *HOMEWORK #7* *due Tue, November 12, 2013*

Written assignment (4 problems).

W1. Let x be a real number such that $|x| < 1$. Compute $\lim_{n\to\infty} \prod_{i=1}^{n} \left(1 + x^{2^i}\right)$.

W2. Define the sequence $\{x_n\}_{n=1}^{\infty}$ by $x_0 = \alpha$, and $x_{n+1} = \frac{e^{x_n}-1}{2}$. Prove the convergence or divergence of the sequence for $\alpha = 0.5$ and $\alpha = 2$.

W3. Let $x_0 = 1$, and $x_{n+1} = x_n + 10^{-10^{x_n}}$ for all $n \geq 1$. What can we say about $\lim\limits_{n\to\infty} x_n$?

W4. Evaluate $\lim\limits_{n\to\infty} \sum_{j=1}^{n} \frac{n}{n^2+j^2}$.

EC. Let $f(x)$ be a positive valued function over the reals such that $f'(x) > f(x)$ for all x. For what k must there exists N such that $f(x) > e^{kx}$ for $x > N$?

Presentation assignment (4 problems).

P1. Find the limit

$$\lim_{n\to\infty} \left(\prod_{k=1}^{n} \left(1 + \frac{k}{n}\right) \right)^{\frac{1}{n}} .$$

P2. For a fixed, positive integer k, the n^{th} derivative of $\frac{1}{x^k-1}$ has the form $\frac{P_n(x)}{(x^k-1)^{n+1}}$, where $P_n(x)$ is a polynomial. Find $P_n(1)$.

P3. For any real number α, we denote by $\{\alpha\}$ the fractional part of α. Consider the sequence $a_n = \{n\sqrt{2}\}$, $n \geq 1$.

1. Show that the sequence a_n contains a subsequence that converges to 0 .
2. Show that the set a_n, $n \geq 1$, is *dense* in the interval $[0, 1]$. (A subset S is called *dense* if for any real number $x \in [0, 1]$, any open interval centered at x contains a number from S. Otherwise said, show that there are terms in $\{a_n\}$ arbitrarily close to any chosen number $x \in [0, 1]$.)

P4. Let $a, b > 0$. Prove that $\lim_{n\to\infty} (a^n + b^n)^{1/n}$ exists and calculate it.