Announcements

• Assigned reading for the week: §3.9 and 3.10.

• HW #6 Due **Thursday, May 12** at 11:00 PM.

• Bring the Week #7 Worksheet **"Sinusoidal Functions and the Piston Problem"** with you to your TA section tomorrow.

• Midterm # 2 Tuesday **May 17**

• HW #7 Due **Thursday, May 19** at 11:00 PM.
  – Do Problems # 1-20 before Tuesday as they cover material that will be on the second Midterm.

Today

• §3.9 Related Rates
Example (§3.9 #3): Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm²?
Example (§3.9 #10): A particle moves along the curve

\[ y = \sqrt{1 + x^3}. \]

As it reaches the point (2,3), the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?
Example (§3.9 #22): A particle is moving along the curve 

\[ y = \sqrt{x}. \]

As the particle passes through the point (4, 2), its \( x \)-coordinate increases at a rate of 3 \( cm/s \). How fast is the distance from the particle to the origin changing at this instant?
Example (§3.9 #22): A particle is moving along the curve

\[ y = \sqrt{x}. \]

As the particle passes through the point \((4, 2)\), its \(x\)-coordinate increases at a rate of 3 \(cm/s\). How fast is the distance from the particle to the origin changing at this instant?

**Solution:** Since the particle is moving, its coordinates are functions of time \(t\).

We write the position at time \(t\) as \((x(t), y(t))\).

Using the distance formula, we see that the distance from the particle to the origin is given by

\[ d(t) = \sqrt{(x(t))^2 + (y(t))^2}. \]

“How fast is the distance from the particle to the origin changing” is asking for the value of the derivative \(d'(t)\), since the derivative measures the instantaneous rate of change of the distance.
Computing the derivative by differentiating (1) implicitly with respect to $t$, we see that

\begin{equation}
(2) \quad d'(t) = \frac{1}{2} \frac{2x(t)x'(t) + 2y(t)y'(t)}{\sqrt{(x(t))^2 + (y(t))^2}}.
\end{equation}

We are told that at the time $t = t_0$ where $(x(t_0), y(t_0)) = (4, 2)$, the “$x$-coordinate increases at a rate of $3 \text{ cm/s}$”. In other words

\[ x'(t_0) = 3 \text{ cm/s}. \]

We need to find the value of $y'(t_0)$. We can find a relationship between $y'(t)$ and $(x(t), y(t))$ and $x'(t)$ by differentiating the defining equation for the curve

\[ y = \sqrt{x}. \]

implicitly with respect to $t$. This gives us

\[ y'(t) = \frac{1}{2} \frac{1}{\sqrt{x(t)}} x'(t). \]

So at the time in question, plugging in $x(t_0) = 4$ and $x'(t_0) = 3$ we see that

\[ y'(t_0) = \frac{1}{2} \frac{1}{\sqrt{4}} 3 = \frac{3}{4}. \]
Plugging these values into equation (2) we get

\[ d'(t_0) = \frac{1}{2} \frac{2x(t_0)x'(t_0) + 2y(t_0)y'(t_0))}{\sqrt{(x(t_0))^2 + (y(t_0))^2}} \]

\[ = \frac{1}{2} \frac{2 \cdot 4 \cdot 3 + 2 \cdot 2 \cdot \frac{3}{4}}{\sqrt{4^2 + 2^2}} \]

\[ = \frac{1}{2} \frac{27}{2 \sqrt{20}} = \frac{27}{4 \sqrt{5}} \approx 3.02 \text{ cm/s}. \]