November 28, 2016

Announcements

- This week 9.1 (Introduction to Differential Equations), 9.3 (Separable Equations)
- Homework # 9A (Center of Mass) & 9B (Separable differential equations) Due Wednesday, November 30, 11:00pm
- Quiz Tomorrow. Tuesday, November 29 (from HW 8A, 8B and/or 9A)
- Printout and bring the Worksheet "DiffEQ.pdf" with you Thursday December 1 for TA sections

Today

- 9.1 Introduction to Differential Equations
- 9.3 Separable Equations

Learning Curves

Psychologists interested in learning theory study **learning curves**.

A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of the training time t. The derivative dP/dt represents the rate at which performance improves.

1. When do you think P increases most rapidly? What happens to dP/dt as t increases?
2. If M is the maximum level of performance of which the learner is capable, what would be a reasonable model for learning?

Learning Curves

Problem From 9.1: Psychologists interested in learning theory study **learning curves**.

A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of the training time t.

The learning process is modeled by the differential equation

$$\frac{dP}{dt} = k(M - P),$$

where M is the maximum level of performance, and k is a positive constant.

Solve this differential equation to find an expression for $P(t)$. What is the limit of this expression?

Solving $\dfrac{dP}{dt} = k(M - P)$

By writing the equation as one involving the differentials dP and dt separately and bringing the P variable to the same side as dP we arrive at

$$\frac{dP}{M - P} = k\,dt.$$

We integrate both sides

$$\int \frac{dP}{M - P} = \int k\,dt \qquad \Longrightarrow \qquad -\ln|M - P| = k\,t + C_0$$

Since $M - P > 0$ we may remove the absolute values and exponentiate both sides to obtain

$$M - P = Ce^{-kt} \qquad \text{or} \qquad P = M - Ce^{-kt}$$

If the initial level of performance at time $t = 0$ is $P_0 = P(0) = M - Ce^0 = M - C$, so $C = M - P_0$, therefore

$$\boxed{P(t) = M - (M - P_0)e^{-kt}}$$

Problem:

1. For what non-zero values of k does the function $y = \sin kt$ satisfy the differential equation $y'' + 9y = 0$?

2. For those values of k, verify that every member of the family of functions
$$y = A \sin kt + B \cos kt$$
is also a solution.

A **separable equation** is a first order differential equation in which the expression for $\frac{dy}{dx}$ can be factored as a function of x times a function of y, i.e.

$$\frac{dy}{dx} = g(x)f(y).$$

If $f(y) \neq 0$, we may write this as

$$\frac{1}{f(y)}\,dy = g(x)\,dx.$$

Integrating on both sides we have

$$\int \frac{1}{f(y)}\,dy = \int g(x)\,dx.$$

If we can evaluate both integrals this yields a "solution" (namely and equation involving x and y without any derivatives). (We may or may not be able to solve for y as a function of x.)

Problem 1 : Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dt} = t\,e^{y}, \qquad y(1) = 0.$$

Problem 2 : Find the solution of the differential equation

$$xy' + y = y^2, \qquad y(1) = -1.$$