# PROBLEM SESSION
# DESCENT TECHNIQUES IN MODULAR REPRESENTATION THEORY

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## 4. LECTURE

Let $k$ be a field of characteristic $p > 0$, and let $G$ be a finite group.

**Problem 4.1.** Show that a $kG$-module $M$ is endotrivial if and only if its restriction $\mathrm{Res}^G_P(M)$ to a $p$-Sylow subgroup $P$ of $G$ is endotrivial.

**Problem 4.2.** Show that the Čech complex is, indeed, a complex. Compute $\check{\mathrm{H}}^0(U, F)$ when $F$ is a sipp sheaf.

**Problem 4.3.** Show that the Čech cohomology group $\check{\mathrm{H}}^1(\mathscr{U}, \mathbb{G}_{\mathrm{m}})$ for the cover $\mathscr{U} = \{G/H \to G/G = *\}$ is naturally isomorphic to the group (with respect to point-wise multiplication) of "weak $H$-homomorphisms $G \to k^\times$", i.e., those functions $u\colon G \to k^\times$ such that:

(a) $u(h) = 1$ for all $h \in H$;
(b) $u(g) = 1$ for all $g \in G$ such that $p \nmid |H^g \cap H|$, and
(c) $u(g_2 g_1) = u(g_2) \cdot u(g_1)$ for all $g_1, g_2 \in G$ with $p \mid |H^{g_2 g_1} \cap H^{g_1} \cap H|$.

**Problem 4.4.** Show that $\check{\mathrm{H}}^0(\mathscr{U}, \mathrm{Pic}^{\mathrm{st}})$ (for the same covering as above) is isomorphic to

$$\{W \in T(P)\} \mid \forall g \in G : \mathrm{Res}^P_{P[g]}(W) \cong {}^g\mathrm{Res}^P_{P[g]}(W) \text{ in } T(P[g])\} \subseteq T(P)$$

where $P[g] := P^g \cap P$.