Math 308 Discussion Problems
Spring 2017

10. Suppose $V = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$, where $\mathbf{u}_1 = (1, 4, 6)$ and $\mathbf{u}_2 = (2, 1, -8)$. Let $\mathcal{B}_1$ denote the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for V, and let $\mathcal{B}_2$ denote a second basis $\{\mathbf{v}_1, \mathbf{v}_2\}$. If $[\mathbf{u}_1]_{\mathcal{B}_2} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}_{\mathcal{B}_2}$ and $[\mathbf{u}_2]_{\mathcal{B}_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}_{\mathcal{B}_2}$, find $\mathbf{v}_1$ and $\mathbf{v}_2$.

11. Let S be a plane in $\mathbf{R}^3$ passing through the origin, so that S is a two-dimensional subspace of $\mathbf{R}^3$. Say that a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ is a *reflection about* S if $T(\mathbf{v}) = \mathbf{v}$ for any vector $\mathbf{v}$ in S and $T(\mathbf{n}) = -\mathbf{n}$ whenever $\mathbf{n}$ is perpendicular to S. Let T be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix

$$\frac{1}{3}\begin{bmatrix} -1 & -2 & 2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

This linear transformation is the reflection about a plane S. Find a basis for S.