Homework 8 **Math 300C Autumn 2015** **Patrick Perkins**

First, do the assigned textbook practice problems:

15.1 (i, ii, iv, vi, ix),
16.1, 16.2, 16.4,
19.2–19.5,
and 20.1, 20.2

1. Prove, for any positive integer n, that 7 divides $6^n + 1$ if and only if n is odd.

2. Prove that, for all integers a and b, $a^2 + b^2 \equiv 0, 1, 2, 4,$ or 5 modulo 8. Deduce that there do not exist integers a and b such that $a^2 + b^2 = 12345790$.

3. Suppose that a positive integer is written as $n = a_k a_{k-1} \ldots a_2 a_1 a_0$ where $0 \leq a_i \leq 9$. Prove that n is divisible by 11 if and only if the alternating sum of its digits,

$$a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^k a_k$$

   is divisible by 11. (Hint: this problem is similar to problem 19.3)

4. Solve the following linear congruences:

   (a) $3x \equiv 15 \pmod{18}$

   (b) $3x \equiv 16 \pmod{18}$

   (c) $4x \equiv 16 \pmod{18}$

   (d) $4x \equiv 14 \pmod{18}$

5. Solve $23x \equiv 16 \pmod{107}$