Homework 6 **Math 300C Autumn 2015** **Patrick Perkins**

NOTE: In problems 2 and 6, please provide a formal proof. For the rest of the questions, your answers should be supported by computations and/or some justification, but no formal proof is needed. I do expect correct notation throughout the entire assignment.

1. From Problems III, page 184, Problem 13:
   Find all the divisors of 126 and 180, and find their greatest common divisor, gcd(126, 180).

2. Use Definition 11.2.1 to prove that if a set of real numbers has a minimum element m, then this element is unique.

3. The set of all possible grades at Random University is $G = \{4.0, 3.9, 3.8, ..., 0.7\} \cup \{0.0\}$. Suppose there are 15 students enrolled in Math 903 at RU. At the end of the term, the instructor must turn in a grade sheet, listing a grade for each student.
   (a) How many different grade sheets are possible for Math 903?
   (b) How many different ways can the instructor for Math 903 assign grades so that no two students receive the same grade?
   (c) The instructor wants to assign a score of 4.0 to exactly 2 students, a score of 3.5 to exactly 5 students, and fail everyone else. How many different ways can the instructor do this?
   (d) Suppose any subset of students in Math 903 can receive a grade of 4.0, from none to the entire class. How many different ways can the instructor select the students who receive 4.0?

4. You have a total of 5 Math books and 4 English books to line up on a shelf.
   (a) How many different ways can you order all the books on the shelf?
   (b) How many different ways can you order them if you want all the Math books to be together and all the English ones to be together?
   (c) How many different ways can you order them if you want just the Math books to be together but the English ones can be wherever.
   (d) How many different ways can you select two Math books and two English books from your collection?

5. a) Compute the coefficient of the $a^{10}b^{15}$ term in the binomial expansion of $(a + b)^{25}$.
   b) Let $n \in \mathbb{Z}^+$. Use the binomial theorem to compute the alternating sum $\displaystyle\sum_{i=0}^{n}(-1)^n\binom{n}{i}$.

6. Is $\sqrt[3]{13}$ a rational number? Prove that your answer is correct.

7. From Problems III, page 186, Problem 25:
   Find the rational number equal to the recurring infinite decimal $2.10012\overline{097}$.