1. Does the Trichotomy law hold for sets? In other words, given two sets and, is it true that one of: $A \subset B$, $B \subset A$ or $A = B$ must be true? If yes, give a proof; if no, give a counterexample.

2. Let $A = \{1, 2, 3, 4\}$.
   (a) List all elements of $\mathcal{P}(A)$ the power set of A.
   (b) List all elements of $A \times A$.

3. Suppose $A \subseteq B$. Is it true that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. If yes, prove it; if no, give a counterexample.

4. Using truth tables, prove the following De Morgan Law for sets: $(A \cap B)^c = A^c \cup B^c$

5. Suppose A, B and C are sets and assume $A \neq \emptyset$. Prove that if $A \times B = A \times C$ then $B = C$. *(Hint: try the contrapositive.)*

6. Give a proof or a counterexample for each of the following statements.
   (a) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x + y > 0$
   (b) $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y > 0$
   (c) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ xy > 0$
   (d) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ xy \geq 0$

   Prove each of the following propositions.

7. Let A and B be sets. Then $A \setminus (A \cap B) = A \setminus B$.

8. Let A, B and C be sets. Then $(A \setminus C) \cap (B \setminus C) = (A \cap B) \setminus C$.

9. Let A, B, and C be sets. Then $(A \cup C) \cap B \subseteq A \cup (B \cap C)$.

10. Let A, B, and C be sets. Suppose $A \cup C \subseteq B \cup C$. Then $A \setminus C \subseteq B$.

11. Let $a \in \mathbb{R}$ and $a \neq 0$. For all integers $n \geq 0$, prove that
$$\sum_{i=0}^{n} a^i = \frac{1 - a^{n+1}}{1 - a}.$$

12. Let a and b be positive integers, with $a > b$. For all integers $n \geq 1$, prove that $a - b$ divides $a^n - b^n$.