1. **Prove that if** \( A, B \) **and** \( C \) **are sets such that** \( C \subseteq A \) **and** \( C \subseteq B \), **then** \( C \subseteq A \cap B \)

2. **For all real numbers** \( x \) **and** \( y \), **prove that** \(|x + y| \leq |x| + |y|\).
   
   You may use the definition of \(|a|\), the addition, multiplication and transitivity laws for inequalities, and all basic arithmetic properties of real numbers.

3. **Consider the following ”proof”**
   
   **THEOREM**: For all sets \( A, B \) **and** \( C \), **if** \( C \subseteq A \cup B \) **and** \( B \cap C = \emptyset \), **then** \( C \subseteq A \).
   
   **”PROOF”**: Let \( A = \{a,b,c,d,e\} \) **and** \( B = \{d,e,f,g\} \). **If** \( C \subseteq A \cup B \), **then** the elements of \( C \) **must be drawn from** **the list** \( a,b,c,d,e,f,g \). **But** \( B \cap C = \emptyset \) **so that** \( B \) **and** \( C \) **have no elements in common**. **Therefore**, the elements of \( C \) **must, in fact**, **be drawn** **from** **the list** \( a,b,c \). **Since each of these elements** **is also an element of** \( A \), **it follows** **that** \( C \subseteq A \).
   
   a) **What is wrong with this argument?**
   
   b) **Write a correct proof of this result.**

4. **Prove that**, for all non-negative integers \( n \), \( 4^{2n+1} + 3^{n+2} \) **is divisible by** \( 13 \).

5. **Show that** \( \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \).

6. **Consider the symbolic statement** \( \forall x \in \mathbb{R}, \forall y \in \mathbb{R}, [(x \leq y) \Rightarrow x^2 \leq y^2] \).
   
   a) **Is the statement true or false?** **If true**, **prove it**. **If false**, **give a counterexample**.
   
   b) **Write the symbolic negation of the statement.**

7. **Draw up a truth table for the statement** \((p \Rightarrow r) \land (r \Rightarrow q)\).

8. **Let propositions** \( S, W, R \) **and** \( T \) **be defined as follows:**
   
   S: The sun shines.
   W: The wind blows.
   R: The rain falls.
   T: The temperature rises.
   
   (i) **Translate into English**: \( \neg(W \land R) \Leftrightarrow S \)
   
   (ii) **Translate into symbols**: ”The sun shines and the wind doesn’t blow, and the temperature rises only if the rain falls.”
   
   (iii) **Suppose all of** \( S, W, R, T \) **are true**. (**Yes, it’s a weird day:)**) **Decide which are true**: a) \( (S \Rightarrow W) \land (\neg R \land T) \)  
   b) \( (S \lor \neg R) \Leftrightarrow (T \lor \neg W) \)  
   c) \( \neg(R \lor \neg T) \land S \)

9. **Prove that** for any sets \( A \) **and** \( B \), \( (A - B) \cap B = \emptyset \).
10. Does the set \( S = \{1 - 1/n \mid n \in \mathbb{Z}^+\} \) have a greatest element? Prove your answer.

11. Let \( f(x) = \sqrt{x + 7} \).

   a) Find its maximal domain \( X \) and list its codomain \( Y \) (in the real numbers).

   b) With the domain and codomain from part a), is \( f \) injective? surjective? Prove your claims.

   c) Write \( f \) as a composition of two functions \( g \) and \( h \), none of which is the identity. Don’t forget to specify the domain and the range of each.

   d) Let

\[
j(x) = \begin{cases} 
  f(x), & \text{if } x \geq 2 \\
  x^2 + c, & \text{if } x < 2.
\end{cases}
\]

For what values of \( c \in \mathbb{R} \) is \( j(x) \) a well-defined function?

12. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) and \( g : \mathbb{R} \rightarrow \mathbb{R} \) be two functions and define \( f + g : \mathbb{R} \rightarrow \mathbb{R} \) by \((f + g)(x) = f(x) + g(x)\). If \( f \) and \( g \) are injective, is \( f + g \) injective? If \( f \) and \( g \) are surjective, is \( f + g \) surjective? If \( f \) is bijective, is \( 2f = f + f \) bijective? Prove your claims, or give counterexamples.

13. Define ”function”. Define ”contrapositive of a statement”. Give examples of each.