YOUR NAME:

FINAL EXAM
Math 300B, Spring 2011

SIGNATURE:

Problem	Points	Score
1	8	
2	10	
3	20	
4	16	
5	10	
6	12	
7	12	
8	12	
Total	100	

• This exam is 1 hr 50 min long. You may use a double-sided sheets of notes & a calculator.

• Provide well-written & complete proofs; make clear which statement follows from another and why.

• On computational problems, unless otherwise stated, please show enough work so I can tell how you are getting your answer.

• Raise your hand if you have questions or if you need more paper.

• Write your work and answers neatly & legibly on the exam paper, in the space provided.

If you must turn in work on the back of pages or on the scratch paper, indicate that you did so (write your name on the scratch paper and make sure it's turned in with your exam.)

$\boxed{1}$ (8=3+5 points)

a) Write out the negation of the statement: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} - \{0\}, xy = 1$.

Negation:

b) Which is true, the original statement S or its negation? Prove your answer.

$\boxed{2}$ (10=8+2 points) Prove carefully using the definitions of subset, set union and set difference that:

For any sets A,B, and C, $(A \cup B) - C \subseteq (A - C) \cup B$.

$\boxed{3}$ (20 = 4 + 6 + 10 points)

a) Define "surjective" function.

<u>Definition:</u> A function $f : X \to Y$ is surjective if and only if...

b) For each of (i.)-(iii.) below, give an example of a function

$$f : \mathbb{Z} \to \mathbb{Z}$$

that has both domain and codomain $\mathbb{Z}$ and satisfies the stated requirements. No proof is needed, just the rule for the function.

i. injective but not surjective: $f(x) =$

ii. bijective (but it's not the identity): $f(x) =$

iii. neither injective, nor surjective: $f(x) =$

c) Prove that if $f : X \to Y$ and $g : Y \to Z$ are two injective functions, then $g \circ f$ is also injective.

$\boxed{4}$ (16 points) Determine the cardinalities of the following sets.

Read each set carefully. Your answer should be either an integer, or one of $\aleph_0$, $\aleph_1$, etc.
You may assume the Continuum Hypothesis: $\aleph_1 = |\mathbb{R}|$, and you need not justify your answers..

a) $A = \mathbb{Q} \times \mathbb{Z} \times \{a, b, c, d\} \times \mathbb{N}$ $\qquad |A| =$

b) $B = \wp(\wp(\{\}))$ $\qquad |B| =$

c) $C = \wp(\wp(\mathbb{R}))$ $\qquad |C| =$

d) $D = \mathrm{Bij}(\mathbb{N}_4, \mathbb{N}_4)$ $\qquad |D| =$

$\boxed{5}$ (10 points) A laboratory study of 50 rabbits showed that 29 liked carrots, 18 liked lettuce, 27 liked bratwurst, 9 liked both carrots and lettuce, 16 liked both carrots and bratwurst, 8 liked both lettuce and bratwurst, and 47 liked at least one of the three foods.

(a) How many rabbits liked all three foods? Explain your answer.

(b) How many rabbits liked none of the three foods? Explain your answer.

$\boxed{6}$ (12 points) Solve the linear congruence below. Give your answer in terms of remainders modulo 244.

(Note that $52 = (2^2)(13)$, $244 = (2^2)(61)$, *and* 61 *is a prime)*

$$52x \equiv 28 \bmod 244$$

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$\boxed{7}$ (12 points) A daycare class consists of 5 girls and 2 boys.

(a) How many ways can the children be arranged in a row for a class photo, with no restrictions?

(b) How many ways can the children be arranged in a row such that the two boys are not together?

(c) How many ways can the teacher choose two girls and one boy to sing a song?

$\boxed{8}$ (12 points) Prove that if n is an odd integer, then 8 divides $n^2 - 1$.

You may use any method you wish, but write a clear and complete proof.