YOUR NAME:

Midterm
Math 300 Spring 2011

$\boxed{1}$ (12 points)

(a) Suppose that P is a true statement, and both R and S are false statements. Which of the following statements are true? (Circle the true ones; no justification needed)

(i) $(P \vee R) \wedge S$

(ii) $S \Rightarrow (P \Rightarrow \neg S)$

(iii) $\neg(R \vee P) \Leftrightarrow S$

(b) Negate the following statement completely (no negative should be left):

$$\forall \epsilon \in \mathbb{R}^+, \exists d \in \mathbb{R}^+ \text{ such that } [\ |x-a| < d \Rightarrow |f(x) - f(a)| < \epsilon\ ].$$

Negation:

(c) Write the contrapositive of the following statement (no negative should be left):

$$[(a \leq b) \wedge (b \leq c)] \Rightarrow (a \leq c)\ .$$

Contrapositive:

$\boxed{2}$ (6 points) Find two major errors in the "proof" below:

Claim: $\frac{1}{x+1} < \frac{1}{x}$, for all real numbers $x \neq 0, -1$.

"Proof":

$$\frac{1}{x+1} < \frac{1}{x}$$
$$\Rightarrow x < x+1$$
$$\Rightarrow 0 < 1$$

This is true, so it follows that $\frac{1}{x+1} < \frac{1}{x}$ for all real numbers $x \neq 0, -1$. QED

$\boxed{3}$ (8 points) Circle the statements that are always true, for any sets A and B (no proof needed):

1. $A \in \mathcal{P}(A)$
2. $A \subseteq \mathcal{P}(A)$
3. $\{A\} \in \mathcal{P}(\mathcal{P}(A))$
4. $A \subseteq A \times B$
5. $(A \cup B)^c = A^c \cup B^c$
6. $B - A \subseteq B$
7. $A \cap B \subseteq A \cup B$
8. $\emptyset \in A$

$\boxed{4}$ (12=8+4 points)
(a) Is the function $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $f(x) = \dfrac{x}{x+1}$ injective? Prove your answer.

(b) Give an example of a function $h : \mathbb{Z} \to \mathbb{Z}$ (notice the domain and codomain!) which is injective but it is not surjective. No proof needed.

$\boxed{5}$ (12 points) Use induction on n to prove that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ for all positive integers n.

Proof: