1 (12 points)
(a) Suppose that $P$ is a true statement, and both $R$ and $S$ are false statements. Which of
the following statements are true? (Circle the true ones; no justification needed)
(i) $(P \lor R) \land S$
(ii) $S \Rightarrow (P \Rightarrow \neg S)$
(iii) $\neg (R \lor P) \Leftrightarrow S$

(b) Negate the following statement completely (no negative should be left):
\[
\forall \epsilon \in \mathbb{R}^+, \exists d \in \mathbb{R}^+ \text{ such that } [\ |x - a| < d \Rightarrow |f(x) - f(a)| < \epsilon ].
\]
Negation:

(c) Write the contrapositive of the following statement (no negative should be left):
\[
[(a \leq b) \land (b \leq c)] \Rightarrow (a \leq c).
\]
Contrapositive:

2 (6 points) Find two major errors in the "proof" below:
Claim: $\frac{1}{x + 1} < \frac{1}{x}$, for all real numbers $x \neq 0, -1$.
"Proof":
\[
\frac{1}{x + 1} < \frac{1}{x} \\Rightarrow x < x + 1 \\Rightarrow 0 < 1
\]
This is true, so it follows that $\frac{1}{x + 1} < \frac{1}{x}$ for all real numbers $x \neq 0, -1$. QED
(8 points) Circle the statements that are always true, for any sets \( A \) and \( B \) (no proof needed):

1. \( A \in \mathcal{P}(A) \)
2. \( A \subseteq \mathcal{P}(A) \)
3. \( \{A\} \in \mathcal{P}(\mathcal{P}(A)) \)
4. \( A \subseteq A \times B \)
5. \( (A \cup B)^c = A^c \cup B^c \)
6. \( B \setminus A \subseteq B \)
7. \( A \cap B \subseteq A \cup B \)
8. \( \emptyset \in A \)

(12=8+4 points)
(a) Is the function \( f : \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R} \), \( f(x) = \frac{x}{x+1} \) injective? Prove your answer.

(b) Give an example of a function \( h : \mathbb{Z} \rightarrow \mathbb{Z} \) (notice the domain and codomain!) which is injective but it is not surjective. No proof needed.
5 (12 points) Use induction on $n$ to prove that $\sum_{i=1}^{n} i^3 = \frac{n^2(n + 1)^2}{4}$ for all positive integers $n$.

Proof: