## Solution to Homework #6, Page 153, Problem 2

There is a unique real number $x$ such that, for every real number $y$, we have $xy + x - 4 = 4y$.

*Proof:* (Existence) Let $x = 4$. Then

$$\begin{aligned} xy + x - 4 &= 4y + 4 - 4 \\ &= 4y \end{aligned}$$

as claimed.

(Uniqueness) Let $z$ be a real number such that $zy + z - 4 = 4y$. Then

$$\begin{aligned} zy + z - 4 &= 4y \\ zy + z &= 4y + 4 \\ z(y + 1) &= 4(y + 1) \end{aligned}$$

Thus if $y \neq -1$ we can conclude that $z = 4 = x$.

Note that if $y = -1$ then any value of $z$ works.

The point is that $z$ has to work for **all** values of $y$. So $z$ must equal 4.