Solution to Homework #2, Page 41, Problem 14(a)

I didn't get anywhere using the propositional calculus to do this. I used truth tables instead.

Let x be an arbitrary element of the universe we are working in. Let P be the statement $x \in A$. Q is $x \in B$ and R is $x \in C$. We will do the problem by showing that

$$[(P \vee Q) \vee R] \wedge \neg[(P \vee Q) \wedge R]$$

is equivalent to

$$([(P \vee R) \wedge \neg(P \wedge R)] \vee [Q \wedge \neg P]) \wedge \neg ([(P \vee R) \wedge \neg(P \wedge R)] \wedge [Q \wedge \neg P])$$

A tedious but routine calculation yields the two truth tables:

P	Q	R	$[(P \vee Q) \vee R] \wedge \neg[(P \vee Q) \wedge R]$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

P	Q	R	$([(P \vee R) \wedge \neg(P \wedge R)] \vee [Q \wedge \neg P]) \wedge \neg ([(P \vee R) \wedge \neg(P \wedge R)] \wedge [Q \wedge \neg P])$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

Since the truth values are the same, the propositions are equivalent. This means that the truth sets are equal and this shows that the sets are equal.