Math 126 **Final Examination** **Autumn 2006**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and put away all electronic devices except your non-graphing calculator.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work on the exam paper, with some explanation in English, if appropriate. Do not do computations in your head. Instead, write them out on the exam paper.
- Give answers in exact form. You do not need to simplify answers algebraically.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the previous page and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	6	
2	8	
3	10	
4	9	
5	8	

Problem	Total Points	Score
6	8	
7	8	
8	9	
9	12	
10	12	
Total	90	

1. [**6 points**] Find the Taylor Series based at $b = 0$ for

$$f(x) = \int_0^x \frac{e^t - 1}{t} dt.$$

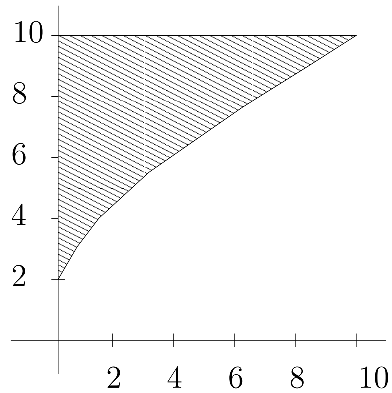
2. (a) [**3 points**] Find the quadratic approximation, $T_2(x)$, based at $b = 1$, for the function

$$f(x) = x \ln x.$$

(b) [**1 point**] Use $T_2(x)$ to approximate $0.9 \ln(0.9)$.

(c) [**4 points**] Using Taylor's inequality, estimate the error of the approximation you obtained in (b).

3. Consider the curve with parametric equations $x(t) = 2t^2 + t$, $y(t) = t^2 + 2t + 2$, for $t \geq 0$.



(a) [**4 points**] Find the equation of the tangent line to the curve at the point corresponding to $t = 1$.

(b) [**6 points**] Find the area in the first quadrant bounded by the curve and the lines $x = 0$ and $y = 10$.

4. Let $f(x, y) = e^{5-x^2-y^2} + 3$.

(a) [**3 points**] Calculate the partial derivatives f_x and f_y.

(b) [**6 points**] Use the linear approximation for f at $(1, 2)$ to estimate the value of y so that $f(1.1, y) = 4.2$.

5. [**8 points**] Find and classify all the critical points of the function $f(x, y) = x^3 + y^2 + 2xy$.

6. (a) [**4 points**] Reverse the order of integration for the integral

$$\int_0^4 \int_{\sqrt{x}}^2 xy \, dy \, dx \, .$$

(b) [**4 points**] Evaluate the integral in part (a). You may integrate either in the original order or in the reversed order.

7. [**8 points**] Find the volume of the region between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ and bounded above by $z = y^2$ and bounded below by $z = 0$.

8. A charged metal ball with mass $m = 0.5$ kg moves in a laboratory. The laboratory is on the space station, so gravity does not affect the motion. The only force acting on the ball is an electric field exerting a constant force of $\mathbf{F} = \langle 2,\ 0,\ 1\rangle$, in units of kg-m/s^2. At $t = 0$ seconds, the particle is at the point (2, 1, 5), coordinates measured in meters, with a velocity of $\mathbf{v}_0 = \langle 0,\ 1,\ 0\rangle$ in m/s.

(a) [**5 points**] Find the position $\mathbf{r}(t)$ of the ball as a function of t. (Recall Newton's second law, $\mathbf{F} = m\mathbf{a}$.)

(b) [**4 points**] At $t = 2$, the electric field is abruptly turned off, so that after that time the force acting on the ball is zero. Find the position of the ball three seconds after the field is turned off.

9. [**12 points**] Consider a triangle ABC in space with vertices $A(1,-2,-1)$, $B(1,1,-1)$, and $C(-3,0,2)$, and the pyramid with base ABC and fourth vertex at the origin O. See Figure 1. Compute the following three quantities. You may find them in any order.

(i) The volume of the pyramid.
Hint: The volume of a pyramid is 1/6 of the volume of the parallelepiped built on the three edges of the pyramid with a common vertex. See Figure 2.

(ii) The area of the triangle ABC.

(iii) The distance from the origin O to the plane containing the triangle ABC.

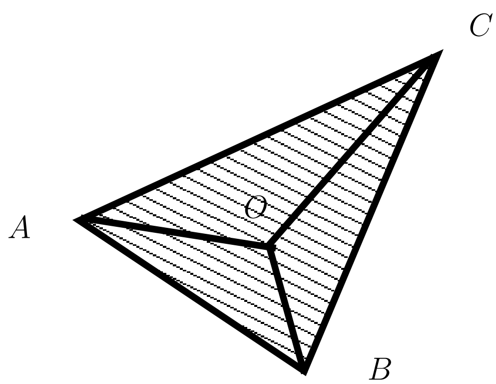


Figure 1

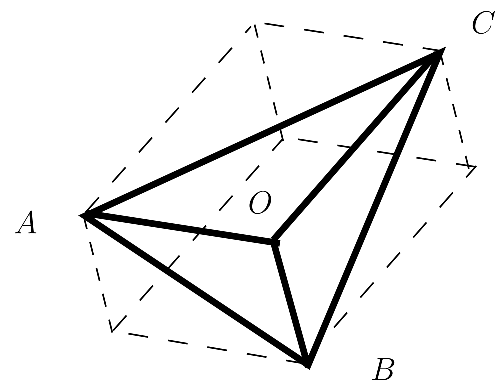


Figure 2

10. Consider a curve lying on the cylinder $x^2 + y^2 = 1$ and given by the vector function $\mathbf{r}(t) = (\cos t, \sin t, t^2)$, for $t \geq 0$.

(a) [**5 points**] Find equations of normal planes at the points $(1, 0, 0)$ and $(1, 0, 4\pi^2)$.

(b) [**2 points**] Find the angle between the two planes from part (a).

(c) [**2 points**] Set up, but do not evaluate, an integral that represents the length of the arc of the curve between the points $(1, 0, 0)$ and $(1, 0, 4\pi^2)$.

(d) [**3 points**] Find $\lim_{t\to\infty} \kappa(t)$, where $\kappa(t)$ is the curvature. Justify your answer either by numerical calculation or geometric observations.