This is a limited open note exam. You may use one (two-sided) page of notes that are in your own handwriting. You may not use books, printed matter, etc. No calculators.

SHOW ALL OF YOUR WORK ON THIS TEST PAPER.
Partial credit will be given for partial solutions, but even correct answers with insufficient or incorrect work will not get much credit. Show enough work on each problem for the grader to tell how you obtained your answer. This may also help you get some partial credit if your answer is incorrect or incomplete. You are responsible for making your solutions readable. Using a few words of English may help the grader understand your work.

If you cannot complete a problem in the given space, then continue your work on the back of the page or on the back of the preceding page. If you do continue your work any place other than the given space for the problem, make sure you note where it is so the grader can find it.

You MUST put your final answer into the answer box for each problem.

Please note: Give all answers as EXACT answers (like π/6 or 1 + \sqrt{2}) unless you are explicitly given directions otherwise.

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1. (12 points total) You MUST show your work to receive any credit.

(a) (6 points) Find the derivative of \( f(x) = 10^{\ln x} \).

\[
\frac{d}{dx} \left( 10^{\ln x} \right) = \frac{d}{dx} (x) = 1
\]

(b) (6 points) Find the indefinite integral \( \int \frac{(x^2 - 1)^{3/2}}{x} \, dx \). (You may assume \( x \geq 1 \)).

\[
\int \frac{(x^2 - 1)^{3/2}}{x} \, dx = \int (x^2 - 1)^{3/2} \, dx = \frac{(x^2 - 1)^{5/2}}{\frac{5}{2}} + C = \frac{2}{5} (x^2 - 1)^{5/2} + C
\]
2. (12 points total)
(a) (6 points) Find the average value of \( f(x) = x\sqrt{1 + x^2} \) on the interval \( 0 \leq x \leq \sqrt{3} \).

(b) (6 points) Evaluate the definite integral \( \int_{1}^{4} \frac{\sqrt{x}}{x + 3\sqrt{x} + 2} \, dx \).
3. (14 points total) The region above the curve $y = \sin x$, below the line $y = 1$, and between the lines $x = 0$ and $x = \pi/2$, is rotated around the line $x = -1$.
(Read that carefully: this region is rotated around the vertical line $x = -1$.)
(a) (4 points) Express the volume of the solid of revolution as a definite integral with respect to $x$. IN THIS PART, DO NOT EVALUATE THE INTEGRAL YET.

(b) (4 points) Express the volume of the solid of revolution as a definite integral with respect to $y$. IN THIS PART, DO NOT EVALUATE THE INTEGRAL YET.

(c) (6 points) Find the volume of the solid of revolution. (Evaluate either integral: your choice.)
4. (14 points total) Consider the region between the lines \( x = 0 \) and \( x = 1 \), above the \( x \)-axis, and below the curve \( y = \frac{1}{\sqrt{4 - x^2}} \).

(a) (4 points) Find the area of this region.

(b) (5 points) Find the \( x \)-coordinate \( \bar{x} \) of the center of mass of this region.

(c) (5 points) Find the \( y \)-coordinate \( \bar{y} \) of the center of mass of this region.
5. (12 points total)

(a) (6 points) Evaluate the limit
\[ \lim_{x \to 0} \frac{2x - \tan^{-1} x}{\sin^{-1}(3x)}. \]

(b) (6 points) Evaluate the limit
\[ \lim_{x \to 0^+} (\sin x)^{1/(\ln x)}. \]
6. (12 points total)
(a) (8 points) Find the indefinite integral \[ \int \frac{2 \ln(x^2 + 1)}{x^3} \, dx. \]

(b) (4 points) Evaluate the improper integral \[ \int_{1}^{\infty} \frac{2 \ln(x^2 + 1)}{x^3} \, dx. \]
7. (12 points) Find the solution $y(x)$ of the initial-value problem

$$x y \frac{dy}{dx} = 2 \ln x, \quad y(1) = 2.$$
8. (12 points) A tank contains 100 liters of pure water. Brine that contains 0.2 kg of salt per liter enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed, and the mixed solution drains from the tank at the same rate of 10 liters per minute. How much salt is in the tank after 30 minutes?