1. Determine if the following limits exist. If they exist, compute them. Justify your answers.

   (a) (4 points) $\lim_{h\to 0}\left(\frac{6}{2h-h^3}-\frac{3}{h}\right)$

$$\begin{aligned}\lim_{h\to 0}\left(\frac{6}{2h-h^3}-\frac{3}{h}\right) &= \lim_{h\to 0}\frac{6-3(2-h^2)}{2h-h^3}\\ &= \lim_{h\to 0}\frac{3h^2}{2h-h^3}\\ &= \lim_{h\to 0}\frac{3h}{2-h^2}\\ &= 0\end{aligned}$$

   (b) (4 points) $\lim_{x\to 2}\frac{2x^2-7x-4}{x-2}$

The numerator is going to -10 *and the denominator is going to* 0*.*
$x-2$ *is negative when* $x<2$ *and positive when* $x>2$*.*
Thus $\lim_{x\to 2^+}\frac{2x^2-7x-4}{x-2}=-\infty$ *and* $\lim_{x\to 2^-}\frac{2x^2-7x-4}{x-2}=\infty$
Since the left and right limits are not equal, the limit does not exist.

   (c) (4 points) $\lim_{x\to -\infty}\frac{\sqrt{5x^4+6}}{x^2-3x}$

$$\begin{aligned}\lim_{x\to -\infty}\frac{\sqrt{5x^4+6}}{x^2-3x} &= \lim_{x\to -\infty}\frac{\sqrt{5x^4+6}\cdot\frac{1}{x^2}}{(x^2-3x)\cdot\frac{1}{x^2}}\\ &= \lim_{x\to -\infty}\frac{\sqrt{5+\frac{6}{x^4}}}{1-\frac{3}{x}}\\ &= \sqrt{5}\end{aligned}$$

2. (7 points) Do not use any differentiation formulas in this problem. Use limits where appropriate. Find the slope of the tangent line to the curve $y = x^2 + 5x$ at the point $(1, 6)$.

Let m be the slope of the tangent line at $x = 1$. Then

$$\begin{aligned} m &= \lim_{h \to 0} \frac{y(1+h) - y(1)}{h} \\ &= \lim_{h \to 0} \frac{(1+h)^2 + 5(1+h) - 6}{h} \\ &= \lim_{h \to 0} \frac{1 + 2h + h^2 + 5 + 5h - 6}{h} \\ &= \lim_{h \to 0} \frac{7h + h^2}{h} \\ &= \lim_{h \to 0} 7 + h \\ &= 7 \end{aligned}$$

3. (7 points) A particle is travelling in a straight line. Its position is given by $x = \left(t^2 - 14\right) e^t$, where x is in feet and t is in seconds. Find all times when the acceleration of the particle is zero.

We must find when the 2nd derivative is zero.

$$\begin{aligned} \frac{dx}{dt} &= \left(t^2 + 2t - 14\right) e^t \\ \frac{d^2x}{dt^2} &= \left(t^2 + 4t - 12\right) e^t \\ 0 &= \left(t^2 + 4t - 12\right) e^t \\ 0 &= t^2 + 4t - 12 \qquad \textit{because } e^t > 0 \\ &= (t+6)(t-2) \end{aligned}$$

Thus the two times are $t = -6$ *and* $t = 2$.

4. (7 points) Let c be a constant. Define $F(x)$ by the piecewise formula

$$F(x) = \begin{cases} cx^2 - 3x & \text{if } x \leq -1; \\ cx + 11 & \text{if } x > -1. \end{cases}$$

Find a value of c that makes F continuous on $(-\infty, \infty)$. Justify your answer. (Your justification should involve limits).

First note that, for any c, $F(x)$ is continuous if $x \neq -1$. Indeed, if $x > -1$ then $F(x)$ is a linear function and if $x < -1$ then $F(x)$ is a quadratic polynomial. These functions are always continuous.

Now consider $x = -1$. We must show that $\lim\limits_{x \to -1} F(x)$ *exists, and equals* $F(-1)$.

Note that $F(-1) = c + 3$

To show that the limit exists, we must show that the left and right limits are equal.

$$\begin{aligned} \lim_{x \to -1^+} F(x) &= \lim_{x \to -1^+} cx + 11 \\ &= 11 - c \\ \lim_{x \to -1^-} F(x) &= \lim_{x \to -1^-} cx^2 - 3x \\ &= c + 3 \end{aligned}$$

Note that the left limit equals $F(-1)$.

Thus it is enough to find a value c so that $11 - c = c + 3$.

$c = 4$ *is the value we want.*

5. (7 points) Calculate the equation of the tangent line to $g(x) = \left|x^2 - 4x\right|$ at $x = 3$.

$$g(x) = \begin{cases} -x^2 + 4x & \text{if } 0 < x < 4; \\ x^2 - 4x & \text{otherwise.} \end{cases}$$

$$g'(x) = \begin{cases} -2x + 4 & \text{if } 0 < x < 4; \\ 2x - 4 & \text{otherwise.} \end{cases}$$

$g'(3) = -2, \qquad g(3) = 3, \qquad y - 3 = -2(x - 3)$

6. (10 points) Find the equations of **all** tangent lines to the curve $y = \dfrac{3x+4}{2x+5}$ that are parallel to the line $7x - 25y = 3$.

Note that the slope of the given line is $m = \dfrac{7}{25}$.

We first find all x values where $\dfrac{dy}{dx} = \dfrac{7}{25}$.

$$\begin{aligned}
\frac{7}{25} &= \frac{dy}{dx} \\
&= \frac{3(2x+5) - 2(3x+4)}{(2x+5)^2} \\
&= \frac{7}{(2x+5)^2} \\
\frac{1}{25} &= \frac{1}{(2x+5)^2} \\
(2x+5)^2 &= 25 \\
2x+5 &= \pm 5
\end{aligned}$$

Thus $x = -5, 0$

Then $y(-5) = \dfrac{11}{5}$ *and* $y(0) = \dfrac{4}{5}$

The lines are $y - \dfrac{11}{5} = \dfrac{7}{25}(x+5)$ *and* $y - \dfrac{4}{5} = \dfrac{7}{25}\,x$