1. (12 total points) Compute the derivatives of the following functions. Do not simplify your answers.

   (a) (4 points) $g(x) = \dfrac{\sin^{-1}\sqrt{3-5x}}{2x+1}$

$$g'(x) = \frac{(2x+1)\cdot \frac{1}{\sqrt{5x-2}} \cdot \frac{-5}{2\sqrt{3-5x}} - 2\cdot \sin^{-1}\sqrt{3-5x}}{(2x+1)^2}$$

   (b) (4 points) $f(t) = t^2 \cdot \sec(3t) \cdot e^{\sqrt{t}}$

$$f'(t) = 2t\cdot \sec(3t)\cdot e^{\sqrt{t}} + t^2 \cdot \sec(3t)\tan(3t)\cdot 3 \cdot e^{\sqrt{t}} + t^2 \cdot \sec(3t)\cdot e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}$$

   (c) (4 points) $y = x^{\sqrt{x}}$

$$\begin{aligned}
\ln y &= \ln x^{\sqrt{x}} \\
&= \sqrt{x}\cdot \ln x \\
\frac{1}{y}\cdot\frac{dy}{dx} &= \frac{1}{2\sqrt{x}}\cdot \ln x + \sqrt{x}\cdot\frac{1}{x} \\
\frac{dy}{dx} &= y\cdot\left(\frac{1}{2\sqrt{x}}\cdot \ln x + \frac{1}{\sqrt{x}}\right) \\
&= x^{\sqrt{x}}\cdot\left(\frac{1}{2\sqrt{x}}\cdot \ln x + \frac{1}{\sqrt{x}}\right)
\end{aligned}$$

2. (8 points) Let $f(x)$ and $g(x)$ be two functions. Suppose we know that $f(3) = 5$ and $f'(3) = -2$. We also know that $g(2) = 3$ and $g'(2) = -1$. Let $h(x) = xf(g(x))$. Calculate $h'(2)$.

$$\begin{aligned}
h'(x) &= 1 \cdot f(g(x)) + x \cdot f'(g(x)) \cdot g'(x) \\
h'(2) &= 1 \cdot f(g(2)) + 2 \cdot f'(g(2)) \cdot g'(2) \\
&= 1 \cdot f(3) + 2 \cdot f'(3) \cdot -1 \\
&= 5 + 2 \cdot -2 \cdot -1 \\
&= 9
\end{aligned}$$

3. (10 points) Find all the points (a, b) on the curve $4x^3 - 6xy + y^2 = 0$ where the tangent line is horizontal.

First compute $\dfrac{dy}{dx}$.

$$\begin{aligned}
\frac{d}{dx}\left(4x^3 - 6xy + y^2\right) &= \frac{d}{dx}0 \\
12x^2 - 6 \cdot 1 \cdot y - 6x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} &= 0 \\
\frac{dy}{dx} &= \frac{6y - 12x^2}{2y - 6x}
\end{aligned}$$

Setting $\dfrac{dy}{dx} = 0$ *we see the tangent is horizontal at the points where the curve* $y = 2x^2$ *intersects the curve* $4x^3 - 6xy + y^2 = 0$.

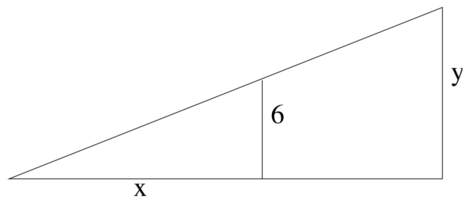
$$\begin{aligned}
4x^3 - 6xy + y^2 &= 0 \\
4x^3 - 6x(2x^2) + (2x^2)^2 &= 0 \\
4x^4 - 8x^3 &= 0 \\
4x^3(x - 2) &= 0
\end{aligned}$$

The possible x-values are $x = 0, 2$.

At $x = 2$ *we have* $4 \cdot 2^3 - 6 \cdot 2 \cdot y + y^2 = 0$. *This gives* $(y - 4)(y - 8) = 0$, *so the points are* $(2, 4)$ *and* $(2, 8)$.

But at $x = 0$ *we get* $y = 0$, *and* $\dfrac{dy}{dx}$ *is undefined at* $(0, 0)$.

4. (10 points) A flashlight is laying on the ground 30 feet from a wall. It is on and pointed straight at the wall. Isobel is walking straight from the wall to the flashlight at a constant speed of 3 feet/second. She is 6 feet tall. How fast is the length of her shadow on the wall increasing when she is 18 feet from the flashlight? Give units in your answer.



Let x be Isobel's distance from the flashlight. Let y be the length of her shadow on the wall.

We know that $\frac{dx}{dt} = -3$*ft/sec. We wish to compute* $\frac{dy}{dt}$ *when* $x = 18$*ft.*

By Similar Triangles we know that $\frac{6}{x} = \frac{y}{30}$

$$\begin{aligned}
\frac{d}{dt}\left(\frac{6}{x}\right) &= \frac{d}{dt}\left(\frac{y}{30}\right) \\
-\frac{6}{x^2}\cdot\frac{dx}{dt} &= \frac{1}{30}\cdot\frac{dy}{dt} \\
-\frac{6}{18^2}\cdot(-3) &= \frac{1}{30}\cdot\frac{dy}{dt} \\
\frac{dy}{dt} &= \frac{5}{3}
\end{aligned}$$

The length of her shadow is increasing at a rate of $\frac{5}{3}$ *ft/sec.*

5. (10 total points) Consider an object moving in the plane whose location at time t seconds is given by the parametric equations:

$$x = 2\cos(2\pi t),\ y = \sqrt{3}\sin(2\pi t)$$

Here $t \geq 0$ and distance is given in meters.

(a) (5 points) Find the speed of the object at time $t = \frac{1}{6}$ seconds.

$$\begin{aligned}
\frac{dx}{dt} &= -4\pi\sin(2\pi t) \\
\frac{dx}{dt}\left(\frac{1}{6}\right) &= -4\pi\sin(2\pi/6) = -2\pi\sqrt{3} \\
\frac{dy}{dt} &= 2\pi\sqrt{3}\cos(2\pi t) \\
\frac{dy}{dt}\left(\frac{1}{6}\right) &= 2\pi\sqrt{3}\cos(2\pi/6) = \pi\sqrt{3} \\
\text{speed} &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \\
&= \sqrt{\left(-2\pi\sqrt{3}\right)^2 + \left(\pi\sqrt{3}\right)^2} \\
&= \pi\sqrt{15}
\end{aligned}$$

The speed is $\pi\sqrt{15}$ *m/sec.*

(b) (5 points) Compute the first time when the horizontal velocity is twice the vertical velocity.

$$\begin{aligned}
\frac{dx}{dt} &= 2\cdot\frac{dy}{dt} \\
-4\pi\sin(2\pi t) &= 2\cdot 2\pi\sqrt{3}\cos(2\pi t) \\
\tan(2\pi t) &= -\sqrt{3} \\
2\pi t &= -\frac{\pi}{3} + k\cdot\pi
\end{aligned}$$

$k = 0$ *gives* $t = -\frac{1}{6}$. *But t cannot be negative.*

Thus the first time is when $k = 1$. Then $2\pi t = \frac{2\pi}{3}$ *and* $t = \frac{1}{3}$.