1. Determine if the following limits exist. If they exist, compute them. Justify your answers.

   (a) (4 points) $\lim_{x \to 2} \frac{x^2 - 4}{2x^2 - 3x - 2}$

   This is a $\frac{0}{0}$ limit. We remove the singularity.

$$\begin{aligned} \lim_{x \to 2} \frac{x^2 - 4}{2x^2 - 3x - 2} &= \lim_{x \to 2} \frac{(x-2)(x+2)}{(2x+1)(x-2)} \\ &= \lim_{x \to 2} \frac{x+2}{2x+1} \\ &= \frac{4}{5} \end{aligned}$$

   (b) (4 points) $\lim_{t \to 0} \left(\frac{5}{3t^2 + 5t} - \frac{1}{t} \right)$

   Combine over a common denominator.

$$\begin{aligned} \lim_{t \to 0} \left(\frac{5}{3t^2 + 5t} - \frac{1}{t} \right) &= \lim_{t \to 0} \left(\frac{5}{3t^2 + 5t} - \frac{1}{t} \cdot \frac{3t+5}{3t+5} \right) \\ &= \lim_{t \to 0} \frac{-3t}{3t^2 + 5t} \\ &= \lim_{t \to 0} \frac{-3}{3t+5} \\ &= -\frac{3}{5} \end{aligned}$$

   (c) (4 points) $\lim_{x \to 2} \frac{\sqrt{3x-2}}{2x^2 - 7x + 6}$

   This is a $\frac{2}{0}$ limit. It is an infinite limit and we must use one-sided limits.
   Factor the denominator to get $2x^2 - 7x + 6 = (x-2)(2x-3)$. Conclude that $2x^2 - 7x + 6$ is positive when $x > 2$ and negative when $\frac{3}{2} < x < 2$.
   Near $x = 2$ the numerator is near 2 and hence is positive.
   Thus $\lim_{x \to 2^+} \frac{\sqrt{3x-2}}{2x^2 - 7x + 6} = \infty$ because the top is positive and the bottom is positive.
   Also $\lim_{x \to 2^-} \frac{\sqrt{3x-2}}{2x^2 - 7x + 6} = -\infty$ because the top is positive and the bottom is negative.
   We conclude $\lim_{x \to 2} \frac{\sqrt{3x-2}}{2x^2 - 7x + 6}$ does not exist.

2. (7 points) Use the limit definition of the derivative on this problem. Find the slope of the tangent line to the curve $y = \sqrt{5x+1}$ at the point $(3,4)$.

Compute the derivative at $x = 3$.

$$\begin{aligned}
\lim_{x\to 3} \frac{\sqrt{5x+1}-\sqrt{5\cdot 3+1}}{x-3} &= \lim_{x\to 3} \frac{\sqrt{5x+1}-4}{x-3}\cdot\frac{\sqrt{5x+1}+4}{\sqrt{5x+1}+4} \\
&= \lim_{x\to 3} \frac{5x+1-16}{(x-3)\left(\sqrt{5x+1}+4\right)} \\
&= \lim_{x\to 3} \frac{5}{\sqrt{5x+1}+4} \\
&= \frac{5}{8}
\end{aligned}$$

The slope of the tangent line is $m_{\tan} = \dfrac{5}{8}$.

3. (7 points) Calculate the equation of the tangent line to $f(t) = 7t^2 \cdot \sin(t)$ at $t = \pi$.

The tangent line is $y - b = m(t-a)$ *where* $a = \pi$, $b = f(\pi) = 0$ *and* $m = f'(\pi)$.

$f'(t) = 14t \cdot \sin(t) + 7t^2 \cdot \cos(t)$

$f'(0) = -7\pi^2$

The equation of the tangent line is $y = -7\pi^2(t-\pi)$

4. (8 points) Let c be a constant. Define $F(x)$ by the piecewise formula

$$F(x) = \begin{cases} cx^2 - 3x & \text{if } x \leq -1; \\ cx + 11 & \text{if } x > -1. \end{cases}$$

Find a value of c that makes F continuous on $(-\infty, \infty)$. Justify your answer. (Your justification should involve limits).

For any value c, $F(x)$ is quadratic if $x < -1$ and linear if $x > 1$. Thus the only value where is might not be continuous is $x = -1$.

We must find c so that $\lim_{x \to -1} F(x) = F(-1)$. Note that $F(-1) = c + 3$.

For the limit to exist, we must have $\lim_{x \to -1^-} F(x) = \lim_{x \to -1^+} F(x)$.

$$\begin{aligned} \lim_{x \to -1^-} F(x) &= \lim_{x \to -1^-} cx^2 - 3x \\ &= c + 3 \\ \lim_{x \to -1^+} F(x) &= \lim_{x \to -1^+} cx + 11 \\ &= 11 - c \end{aligned}$$

Thus we must have $c + 3 = 11 - c$, or $c = 4$.

This makes F continuous everywhere because $\lim_{x \to -1^-} F(x) = F(-1)$.

5. (8 points) Find **two** different points on the curve $y = x^3$ at which the tangent line passes through the point $(-2, 0)$.

Let (a, b) be a point on the curve with the given property.

The tangent line looks like $y - b = m(x - a)$. We have $b = a^3$ and $x = -2$, $y = 0$.

$m = \left. \dfrac{dy}{dx} \right|_{x=a} = 3a^2$

$$\begin{aligned} 0 - a^3 &= 3a^2(-2 - a) \\ 2a^3 + 6a^2 &= 0 \\ 2a^2(a + 3) &= 0 \\ a &= -3, 0 \end{aligned}$$

The two points are $(0, 0)$ and $(-3, -27)$.

6. (8 points) Find the equations of **all** tangent lines to the curve $y = \dfrac{5x-4}{2x-3}$ that are parallel to the line $7x+9y=2$.

   First compute $\dfrac{dy}{dx} = -\dfrac{7}{(2x-3)^2}$

   The slope of $7x+9y=2$ *is* $m = -\dfrac{7}{9}$.

   Solve $-\dfrac{7}{(2x-3)^2} = -\dfrac{7}{9}$ *to get* $x = 0, 3$.

   The points on the curve are $(0, 4/3)$ *and* $(3, 11/3)$.

   The lines are $y - \dfrac{4}{3} = -\dfrac{7}{9}x$ *and* $y - \dfrac{11}{3} = -\dfrac{7}{9}(x-3)$.